Stability and complexity in ecological systems

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Abstract

The paper attempts to answer an outstanding question in theoretical ecology whether structural complexity is essential for dynamical complexity to exist. Advances in dynamical systems theory and their application to ecosystem analysis has enabled us to have a better grasp of the concept of dynamical complexity. Our basin boundary calculations indicate that structural complexity is not necessary for dynamical complexity to exist. Very simple ecosystems can display dynamical behaviour which is unpredictable in certain situations. In certain cases, when riddled basins are found, even qualitative predictability is denied.

1. Introduction

One of the important questions in theoretical ecology concerns with the determination of a relationship between stability and complexity of an ecosystem. There exist primarily two different approaches to address this question:

(i) Dynamical system approach,
(ii) Statistical–mechanical approach.

The first approach was introduced by May [1] and was subsequently developed by Schaffer and Kot [2,3]. Recently, this approach was used by the authors [4] to study whether chaos is an ecological reality. This approach is based on the hope that the essential properties of the subsystems are not altered by their larger settings. This approach involves construction of model systems to represent these subsystems. These model systems often involve nonlinear evolution equations. The dynamical behaviour of these systems is explored in order to have an idea about the parameter regimes displaying different dynamical possibilities (different types of attractors in the phase space). The second approach [5,6] relies on the existence of conserved quantities e.g., Hamiltonian, angular momentum, etc. The nonexistence of such conserved quantities for ecological systems has been a major handicap in the practice of this approach. The puzzling issue arising out of these two distinct approaches has been the contradictory results obtained from them. The dynamical system approach suggests that even simple ecosystems could be stable. The second approach indicates that complexity (a large number of species and interaction) is essential to guarantee ecological stability. Due to the advances made in the study of the dynamical system approach, it is realised that the concept of ecological stability has become more intricate and sophisticated. The understanding of complexity [7] has also improved a great deal during recent years. Studies are being made to understand the relationship between stability and complexity.
In this paper we attempt to answer the question: Is complexity necessary for the stability of an ecosystem? While answering this question we may also need the answers to the following: (a) Does the structural complexity guarantee structural stability? (b) What is the relationship between structural stability and dynamic complexity?

In Section 2, we present the methodology used. In Section 3, an attempt is made to give a meaning to the stability of an ecosystem. Numerical results are also presented in this section. The issue of complexity is discussed in Section 4. The relationship between stability and complexity is presented in Section 5. Conclusions are given in Section 6.

2. Methodology

In [8], two of the present authors gave a method to find meaningful parametric values (for the parameters appearing in the model systems) for numerical calculations. We present this method briefly to understand the technique that is being used in the present paper. To examine the relationship between stability and complexity we have used two model systems which were presented in [8]. The parameter regimes displaying different dynamical possibilities are identified. The calculation of basin boundaries of coexisting attractors is particularly useful in addressing the issues raised in this paper.

2.1. Selection of parametric values for dynamical study

To our knowledge, there exists no systematic procedure which helps us to make choices of biologically realistic parametric values. The selection of the parametric values must be guided by the biological principles. In what follows, we describe a method for the dynamical study of three-species ecosystems. The species are connected with each other in a food-chain relationship. The top prey and the middle predator give a biologically meaningful subsystem. Let $X$, $Y$, $Z$ be the top prey, specialist predator and generalist predator, respectively. In order to be a biologically meaningful system a subsystem should qualify as a Kolmogorov (K) system [1]. The last term in Eq. (6b) (see Section 3) is omitted to get a subsystem. Application of the Kolmogorov theorem shows that this subsystem will qualify as a K-system when

\begin{align*}
w_3 &> a_3, \quad (1a) \\
ad \frac{a_1}{b_1} &> \frac{D_1 a_2}{(w_1 - a_2)} (1b)
\end{align*}

Suppose that there exists a prey $\tilde{X}$ for predator $Z$ other than $Y$. Let $A$ be the rate of self-reproduction for this prey and $K$ be the carrying capacity of its environment. In the Leslie–Gower scheme the growth rate equations for the two populations are

\begin{align*}
\frac{d\tilde{X}}{dt} &= A\tilde{X} \left(1 - \frac{\tilde{X}}{K}\right) - \frac{B\tilde{X}Z}{(\tilde{X} + E)}, \quad (2a) \\
\frac{dZ}{dt} &= cZ^2 - \frac{w_3 Z^2}{(\tilde{X} + D_3)}, \quad (2b)
\end{align*}

For this subsystem to be a K-system, the following inequalities are to be satisfied:

\begin{align*}
\frac{B\tilde{X}Z}{(\tilde{X} + E)^2} &< \frac{BZ}{(\tilde{X} + E)} + \frac{A\tilde{X}}{K}, \quad (3a) \\
c(\tilde{X} + D_3) &- w_3 < 0. \quad (3b)
\end{align*}
A Kolmogorov system can either admit a stable equilibrium or a stable limit cycle solution. The subsystem given by Eqs. (2a) and (2b) would exhibit a stable equilibrium when the following two inequalities are satisfied:

\[ A + 2cZ^* < \frac{2Ax^*}{K} + \frac{2w_2Z^*}{(x^* + D_3)^2} + \frac{BEZ^*}{(x^* + E)^2}, \]  

\[ \left( A - \frac{2Ax^*}{K} + \frac{BEZ^*}{(x^* + E)^2} \right) \left( 2cZ^* - \frac{2w_2Z^*}{(x^* + D_3)^2} \right) + \frac{Bw_2x^*Z^2}{(x^* + E)(x^* + D_3)^2} > 0, \]  

where \( x^* \) and \( Z^* \) are the equilibrium populations.

When the values of the parameters of this subsystem are chosen in such a way that the constraints (3a) and (3b) are satisfied and the inequalities (4a) and (4b) are violated, the subsystem admits a limit cycle solution. We find that for the following values of the parameters the subsystem has a limit cycle solution:

\[ A = 1, \quad K = 50, \quad B = 1, \quad E = 20, \quad c = 0.0062, \quad w_2 = 0.2, \quad D_3 = 20. \]

There may exist other sets of values satisfying the above criteria. The first subsystem would admit limit cycle solutions when the Kolmogorov conditions (1a) and (1b) are satisfied and the conditions from the linear stability analysis are violated. The linear stability conditions are

\[ D_1 > 0, \]  

\[ 2b_1 \left( \frac{a_2D_1}{w_1 - a_1} \right) + b_1D - a_1 > 0. \]

We find that for the following set of parametric values limit cycle solutions exist:

\[ a_1 = 2, \quad b_1 = 0.05, \quad D = 10, \quad a_2 = 1, \quad w_1 = 2, \quad D_1 = 10. \]

We choose the maximum value of \( w = 1 \) as it is the maximum per capita removal rate of prey population. Again, there may exist other sets of values satisfying the above criteria.

We now try to link the two subsystems. One possible way which is biologically sound is shown in Fig. 1. The linking scheme would depend on how the individual populations of the two subsystems are related with each other. This link scheme can be mathematically represented by adding a term \(-w_2YZ/(Y + D_2)\) to the second equation of the first subsystem. This gives us the original Eq. (6b). Since the meaning of \( D_1 \) and \( D_2 \) are the same, and \( D_2 \) is not an important parameter as far as asymptotic dynamics of the complete system is concerned, \( D_2 \) can also be assigned numerical values in the range of \( D_1 \). On the other hand, \( w_2 \) can be varied from 0.1 to 1.

The most crucial part of the present methodology is the following conjecture: A model ecological system would either exhibit limit cycle solutions or chaos when both the corresponding subsystems are in the oscillatory mode.

The selection of the parametric values for the original system is done by omitting those appearing in the growth equation of the pseudoprey (\( \hat{x} \)). In our case, the set of parameter values for which the system admits limit cycle solution is found to be

![Diagram](image_url)

Fig. 1. Relationship between food-chain species and pseudo-prey.
\[ a_1 = 2, \quad b_1 = 0.05, \quad w = 1, \quad D = 10, \quad a_2 = 1, \quad w_1 = 2, \quad D_1 = 10, \quad w_2 = 0.3, \quad D_2 = 10, \quad c = 0.0062, \quad w_3 = 1, \quad D_3 = 20. \]

There may exist other sets of parameter values which satisfy the above criteria. The rationale for selection of \( w_2 \) is based on the fact that it plays a role similar to that of \( w \).

2.2. Calculation of the basin boundary structures

For a dynamical behaviour to be of any practical value it is essential that it should exist in a wide parameter range and the corresponding natural measure in 2D parameter scan should be nonzero. In addition, it must fulfill the requirement that it should possess a phase space of initial conditions whose natural measure (area or volume) is nonzero. When these two conditions are met by a dynamical system for a particular dynamical behaviour, then the same is understood to be a robust one and is considered to have some significance. In order to obtain the information how a given system would behave when acted upon by external disturbances/perturbations, it is necessary to know the structure of the basin boundaries of coexisting attractors (which is a common occurrence in nonlinear dynamical systems). In what follows we would mention briefly how basin boundary calculations are performed in actual practice.

We shall first define the basin of infinity. Let SD denote the diameter of the computer screen. It may be possible that the point at infinity is an attractor. Since we cannot examine rigorously whether the trajectory of a point goes to infinity, we conclude that a trajectory diverges or is diverging if it leaves the computer screen area, that is, it goes to left or to right of the screen by more than one SD width of the screen, or goes above or below the screen area by more than one SD screen height. The basin of infinity is the set of initial points whose trajectories are diverging. Maryland Chaos group has done pioneering work in this area and have developed a tool to calculate basin boundary structures. We have used the research version of the software which accompanied the book entitled "Dynamics: Numerical Explorations" authored by Nusse and Yorke [9]. We have used the basins and attractors structure (BAS) method for all the computations. This method divides the basin into two groups, first the (generalized) basin of attraction A whose points will be plotted, and second the (generalized) basin B whose points will not be plotted. A generalized attractor is the union of finitely many attractors, and a generalized basin is the basin of a generalized attractor. The routine BAS does not plot bowl lying outside. It considers a 100 \times 100 grid of boxes covering the screen. Its strategy is to test each grid point which is the centre of the grid box. In the event that the centre of a grid box is in basin A, then the same is plotted (coloured). In the default case, basin A is the set of points whose trajectories are diverging, while basin B is empty. Therefore, BAS routine will plot a grid box if the trajectory of its centre is diverging. The important aspects of the basin boundary calculations are to specify the basins \( A \) and \( B \) and to find the radius \( R_A \), where \( R_A \) stands for radius of attraction for storage vectors which help to specify the basins A and B. The value of \( R_A \) will be different for different dynamical systems. It must be set appropriately in order to avoid any misleading basin picture.

3. The meaning of stability

Stability is an important attribute of an ecological system. We normally discuss two types of stabilities: (i) structural and (ii) dynamic. Ecological processes like species invasion or diseases may cause vital changes in the structural arrangements of an ecosystem. These structural changes may alter the dynamic regimes of the concerned ecological system altogether. In this paper, we shall not discuss structural stability but concentrate on various aspects of dynamical stability.

An ecosystem is said to be stable if it displays a particular type of attractor in its phase space in wide parametric regimes. This definition of stability is incomplete when we consider the phenomenon of bistability/multistability, very commonly observed in the dynamical behaviour of model ecological systems. In this case, stability demands that the basin boundary of the coexisting attractors should be smooth (non-fractal). If the basin boundaries are fractal, then external perturbations would lead to unpredictable outcomes. For systems which possess smooth basin boundaries [17] with one of the attractors existing in
narrow parameter regimes, it is difficult to discuss its stability. The model systems studied in this paper exhibit such behaviour [8].

Consider the following two three species model systems (see [8]):

Model system I:

\[
\frac{dX}{dt} = a_1 X - b_1 X^2 - \frac{w_1 X Y}{(X + D_1)},
\]

\[
\frac{dY}{dt} = -a_2 Y + \frac{w_1 X Y}{(X + D_1)} - \frac{w_2 Y Z}{(Y + D_2)},
\]

\[
\frac{dZ}{dt} = c Z^2 - \frac{w_3 Z^2}{(Y + D_3)}.
\]

Model system II:

\[
\frac{dX}{dt} = a_1 X - b_1 X^2 - \frac{w_1 X Y}{(X + D)},
\]

\[
\frac{dY}{dt} = -a_2 Y + \frac{w_1 X Y}{(X + D_1)} - \frac{w_2 Y^2 Z}{(Y^2 + D_2)},
\]

\[
\frac{dZ}{dt} = c Z^2 - \frac{w_3 Z^2}{(Y + D_3)}.
\]

The above two systems represent an ecological situation where a top prey \( X \) is predated by \( Y \) (invertebrate in case of system I and vertebrate in case of system II) which, in turn, is predated by \( Z \). The common property of both the systems is that predators \( Y \) and \( Z \) are specialist and generalist, respectively.

For the discussion to be complete, we reproduce Tables 1 and 2 of [8], which are the main results of dynamical study of these model food-chains. It can be observed that these systems support stable equilibrium point and stable limit cycle attractors in reasonably large parameter regimes, whereas, chaotic behaviour is exhibited in narrow parameter regimes. As the model systems are approximate representations of reality, it is necessary to study the nature of basin boundaries. The nature of these boundaries gives us insight into how a system would behave in fluctuating environments. With this in mind we have computed the basin boundaries for typical limit cycle and chaotic attractors in both the systems. Figs. 2–5 present the

<table>
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<th>Parameter kept constant</th>
<th>Parameter varied</th>
<th>Range in which parameter was varied</th>
<th>Dynamical outcome</th>
</tr>
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<tr>
<td>( b_1, w_2, c )</td>
<td>( a_1 )</td>
<td>1.5–1.7</td>
<td>Stable focus</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.8–1.95</td>
<td>Stable limit cycle</td>
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<td></td>
<td></td>
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<td>Strange chaotic attractor</td>
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<tr>
<td></td>
<td></td>
<td>2.05–2.5</td>
<td>Stable limit cycle</td>
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<tr>
<td>( a_1, w_2, c )</td>
<td>( b_1 )</td>
<td>0.035–0.048</td>
<td>Stable limit cycle</td>
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<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>Strange chaotic attractor</td>
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<td></td>
<td></td>
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<td>Stable limit cycle</td>
</tr>
<tr>
<td>( a_1, b_1, c )</td>
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<td></td>
<td></td>
<td>0.3</td>
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<td></td>
<td></td>
<td>0.5</td>
<td>Limit cycle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.55–1.0</td>
<td>Strange chaotic attractor</td>
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<tr>
<td></td>
<td></td>
<td>0.003–0.0255</td>
<td>Stable limit cycle</td>
</tr>
<tr>
<td></td>
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<td>0.026–0.045</td>
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<td></td>
<td></td>
<td>0.05</td>
<td>Stable focus</td>
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\*The values at which parameters \( a_1, b_1, w_2 \) and \( c \) were kept constant are: \( a_1 = 2, b_1 = 0.05, w_2 = 0.55 \) and \( c = 0.0257 \).
Table 2
Simulation experiments of model II with parameter values $w = 1$, $D = 10$, $a_2 = 1$, $w_1 = 2$, $D_1 = 10$, $D_2 = 100$, $D_3 = 20$, $w_3 = 1^a$

<table>
<thead>
<tr>
<th>Parameter kept constant</th>
<th>Parameter varied</th>
<th>Range in which parameter was varied</th>
<th>Dynamical outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$, $w_2$, $c$</td>
<td>$a_1$</td>
<td>0.5-1.5</td>
<td>Stable focus</td>
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<tr>
<td></td>
<td></td>
<td>2.0</td>
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<td>2.2-2.5</td>
<td>Stable limit cycle</td>
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<tr>
<td>$a_2$, $w_2$, $c$</td>
<td>$b_1$</td>
<td>0.03-0.04</td>
<td>Stable limit cycle</td>
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<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>Strange chaotic attractor</td>
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<td></td>
<td></td>
<td>0.06</td>
<td>Stable limit cycle</td>
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<tr>
<td></td>
<td></td>
<td>0.08-0.1</td>
<td>Stable focus</td>
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<tr>
<td>$a_2$, $b_1$, $c$</td>
<td>$w_2$</td>
<td>0.055-1.3</td>
<td>Stable limit cycle</td>
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<td>1.45-1.6</td>
<td>Strange chaotic attractor</td>
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<td>1.65-1.7</td>
<td>Stable limit cycle</td>
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<tr>
<td>$a_2$, $b_1$, $w_2$</td>
<td>$c$</td>
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<td></td>
<td>0.03</td>
<td>Stable limit cycle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.04</td>
<td>Stable focus</td>
</tr>
</tbody>
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*a* The values at which parameters $a_1$, $b_1$, $w_2$ and $c$ were kept constant are: $a_2 = 2$, $b_1 = 0.05$, $w_2 = 1.45$ and $c = 0.0257$.

Fig. 2. (a) $XY$ view of the basin boundary structure for the chaotic attractor in system I for $a_1 = 2$, $c = 0.0257$, $w_2 = 0.55$, $b_1 = 0.05$, $w = 1$, $D = 10$, $a_2 = 1$, $w_1 = 2$, $D_1 = 10$, $D_2 = 10$, $D_3 = 20$ and $w_3 = 1$. (b) $YZ$ view of the basin boundary structure for the chaotic attractor in system I for $a_1 = 2$, $c = 0.0257$, $w_2 = 0.55$, $b_1 = 0.05$, $w_1 = 2$, $D = 10$, $a_3 = 1$, $D_1 = 10$, $D_2 = 10$, $D_3 = 20$ and $w_3 = 1$.

2D projections of the basin boundaries for the two attractors. In the colour figures, the meanings of the various colours are as follows:

- Dark blue: colour of points that diverge from the screen area,
- Sky blue: basin of first attractor,
- Green: colour of first attractor,
- Maroon: basin of second attractor,
- Pink: colour of second attractor,
- Black: colour of the screen (not the menu colour), that is, the hole created by the system.

Fig. 2a gives the $XY$ view of the chaotic attractor in system I at $a_1 = 2$, $c = 0.0257$ and $w_2 = 0.55$. The interesting feature we observe from this diagram is the co-existence of a repeller with the chaotic attractor. The repeller repels the system's trajectories to infinity. This signifies that the outcome of a simulation experiment with initial condition from the basin of repeller is physically unrealisable, but its true signifi-
cance can be realised when one considers system response to disturbances (e.g., species invasion, drought, fire, etc.). In such cases, system dynamics would be dominated by trajectories diverging towards infinity. For a few moments the repeller would occasionally land up on the chaotic attractor because of the presence of holes in the basin of the repeller. Fig. 2b provides the YZ view of the basin boundary structure. This reveals the presence of additional features in the basin boundary diagram. We observe riddled basins \([10,11]\) for a line attractor lying in the \(Y\) direction which coexists with the chaotic attractor. In this case, each basin of attraction is full of holes at arbitrarily small scales. A natural outcome of this geometry is that the final state of the system (attractor) cannot be predicted with certainty if there is any error in the measurement of the initial condition. Since these errors are unavoidable, riddled basins always lead to unpredictable outcomes. The interesting feature to note here is that the riddled basin lies in the basin of the
repeller which has many rectangular and square holes created by basins of line and chaotic attractors. This complicated basin boundary structure suggests that the system dynamics may have loss of even qualitative predictability in case of external disturbances. In such situations, the concept of dynamical stability will lose its meaning. As pointed out earlier, chaotic behaviour is observed in very narrow parameter regimes. This observation along with the conclusion drawn from the basin boundary diagrams suggest that the common notion of dynamical stability has a very restricted practical value. A complete discussion of dynamical stability of model systems must involve basin boundary structures of coexisting attractors (repellers).

The \( XY \) view of the basin boundary structure for the chaotic attractor of the second system found for \( a_1 = 2, D_2 = 100, \ c = 0.0257 \) and \( w_2 = 1.45 \) is presented in Fig. 3a. The repellers basin has a large hole created by the basin of the chaotic attractor. The basin of the repeller dominates over that of the chaotic attractor. This means that the system, for these parameter values, would behave in a meaningless way. This suggests that the ecological system would be rendered nonfunctional even by relatively small amplitude disturbances. The associated \( YZ \) view is given in Fig. 3b. A line attractor coexists with the chaotic attractor and a dominating repeller. Numerous holes are created in the repeller basin by the basin of the line attractor. Few holes created by basins of the chaotic attractors and by an invisible attractor are also observed. As in the case of system I, this suggests that a consideration of basin boundary structure is indispensable as far as discussion of ecological stability is concerned. Specifically, in this case, the dynamical behaviour of the system, in the midst of a disturbance, would be difficult to predict as numerous holes make their presence felt on the body of the repeller.

Let us now study the basin boundary structures for limit cycle attractors. We have noted earlier that the limit cycle behaviour exists in considerably large parameter regimes in both the model systems. Fig. 4a shows the \( XY \) view of the basin boundary structure for the limit cycle attractor in system I. A dominant repeller coexists with the limit cycle attractor for the parameter values \( a_1 = 1.8, \ c = 0.0257 \) and \( w_2 = 0.55 \) (the values of the rest of the parameters are as given in Table 1). The basin of repeller has a large hole created by the basin of limit cycle attractor. In the case of small amplitude disturbances, the system's dynamics would be confined to the limit cycle attractor. For strong external perturbations (exemplified by natural disturbances) the ecosystem might stop functioning. The corresponding \( YZ \) view is shown in Fig. 4b. A distinct feature of this basin boundary diagram is the presence of an invisible attractor which has riddled basin boundary with limit cycle and line attractors. The interesting aspect of this boundary structure is that the riddled basin lies over the basin of the repeller which has holes created by the basin of line and limit cycle attractors. The basin structure suggests that in the event of disturbances, the system behaviour would be of mixed nature. In certain situations, the dynamical behaviour of the entire system
may depend only on the limit cycle attractor. In other situations, it may often be either unpredictable or meaningless depending on the strength of disturbances. The X,Y view of the limit cycle attractor at \( a_1 = 2.2, c = 0.0257 \) and \( w_2 = 0.55 \) is given in Fig. 5a. The limit cycle has two stable oscillations of different periods and is observed in reasonably wide parameter regimes. The basin boundary of the limit cycle attractor with the chaotic attractor is smooth, but the repeller is dominant. The basin of the repeller has a large hole created by the limit cycle attractor. This indicates that the system trajectories would unfold themselves on the limit cycle attractor for some time before being taken away by a large disturbance. The corresponding Y,Z view is presented in Fig. 5b. The characteristic feature of the basin boundary structure is the presence of numerous holes created by the basin of the limit cycle and the line attractors. In the case of a medium scale disturbance system's behaviour would be predictable, and in this case, trajectories would lie on either of the two attractors.

4. The meaning of complexity

There exist two types of complexities in relation to ecological systems: (i) structural and (ii) dynamical complexity. The first is concerned with how individual species are interconnected with each other, that is, with the interactions linking constituent species of an ecosystem. The second gives us an idea how an ecosystem might behave when perturbed by a disturbance of known strength and nature. It arises mainly because of the coexistence of two or more dynamical possibilities at the same set of parametric values. Then, in this case, the system's dynamics jumps from one attractor to the other when perturbed by an external disturbance. The frequency of these jumps and the residence times of the system's trajectories on any of the coexisting attractors is decided by the exact details of the basin boundary structure. Basin boundary structures provide a qualitative idea of the dynamical complexity of a system. A system's response to external disturbances depends upon its basin boundary structure. In case of smooth basin boundaries one would expect that the system behaves predictably. On the other hand it is very difficult to predict the final state of a dynamical system with fractal basin boundaries [12,13].

A structurally simple system may have a complex dynamical behaviour whereas a structurally complex system might be dynamically simple. The dynamical complexity of simple ecological systems is well known to theoretical ecologists [8,14,15]. Structurally complex ecosystems are the ones which also possess greater species diversity. Tilman and Downing [18] have concluded that more diverse plant communities are more stable. Here, stability refers to resistance and resilience combined. Their conclusion was based on a long-term field study of grasslands. A similar conclusion has recently been drawn by Naeem and Li [19] who have carried out microcosm experiments involving algae (producers) and bacteria (consumers). These studies suggest that structural complexity is indispensable for ecosystem stability as measured by the yardsticks of reliability and predictability.

5. The relationship

The concept of stability has only dynamical connotation for the present paper. There is direct relationship between stability and complexity of ecological systems. A dynamically stable system is the one which displays a particular type of dynamical behaviour in wide parameter regimes. Throughout this regime it must not possess dynamical complexity. In the event that it has dynamical complexity, the system behaviour when acted upon by a disturbance may not be stable at all.

Our computations and discussions in the previous sections shows that structural complexity is not essential for dynamical complexity to exist. Of course, it is an intuitive understanding that structural complexity guarantees structural stability, e.g., it is very unlikely that a structurally complex ecosystem will be invaded by foreign species. Whenever it occurs, large structural changes in the ecosystem are very less probable. However, structural stability does not guarantee that a system would be dynamically stable also. Changes in physical parameters caused by changes in physiological conditions would be sufficient to bring about unpredictable changes in the dynamics of structurally stable system.
6. Conclusions

In this paper, we have attempted to answer the question whether dynamical complexity [7] is possible without structural complexity. Our simulation experiments suggest that the answer is a definite 'yes'. The relationship between dynamical stability and structural complexity has been a contentious issue. Advances in dynamical systems theory [16,17] have enabled us to understand dynamical complexity in a more general context. Our results obtained by the dynamical study of the model systems suggest that structurally simple systems may be dynamically very complex in certain situations. On the other hand, there is no guarantee that a structurally complex system would be dynamically complex also. It may be necessary to carry out microcosm experiments to test whether structurally complex systems are also structurally stable.

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