IV. Frequency Limitations of the Filter

In this section, the actual values of W0 and Q of the filter are derived, taking into effect the rolloff of the OA gain. Assuming that ideal OA’s are used for A1 and A2 such as the pA741,

\[ A_{\infty} = \frac{GB}{s + \omega_0} \]  

(24)

where we have the following.

A. Open-loop dc gain of the OA.

\[ \omega > \omega_0 \]  

Open-loop 3-dB bandwidth of the OA.

\[ GB \]  

Gain-bandwidth product of the OA.

Substituting from (6), (7), (9), (10), (24), and (25) in (1), and after some approximations, one gets the following expression for the denominator of the transfer function:

\[ D(s) = P_1(s) + \omega_0^2 GB P_2(s) \]  

(26)

where

\[ P_1(s) = \frac{s}{s + 1} \]  

(27)

and

\[ P_2(s) = \left[ s + \frac{1}{Q^2 + 1} \right] \left( \frac{Q + \frac{1}{\omega_0}}{Q + \frac{1}{\omega_0}} \right) \]  

(28)

Thus the actual values of \( w_0 \) and Q are

\[ \omega_0 = \frac{\omega_0}{w_0} \]  

(29)

According to the Budak-Petrela technique [31], it follows that

\[ \frac{\Delta \omega_0}{\omega_0} = -\frac{\omega_0}{Q \ GB} \]  

(30)

\[ \frac{AQ}{Q} = \frac{s}{s + 1} \left( \frac{1}{Q^2 + 1} \right) \frac{\omega_0}{GB} \]  

(31)

Thus the actual values of w0 and Q are

\[ \alpha = \alpha_w \left[ 1 - \frac{\omega_0}{Q \ GB} \right] \]  

(32)

\[ \beta = \beta_w \left[ 1 + \frac{Q}{\omega_0} \right] \]  

(33)

Equation (30) gives the frequency limitation of the filter for a specified \( \omega_0 \) and a given Q. For example, for Q = 1, and for a specified maximum allowable change in \( w_0 \) and using the same OA, it can be seen that the given filter realizes \( W_0 \) which is twice that realizable using either the negative K Sallen-Key filter or the positive K Sallen-Key filter having equal R and equal C.

V. Stability Analysis

From (26), (27), and (28), and applying Routh’s criterion for stability, it is found that for stability

\[ \frac{2 \ \omega_0}{Q \ GB} + \frac{6 \ \omega_0}{Q \ GB} + \frac{s \ \omega_0}{Q} > Q^2. \]  

(34)

That is, for \( Q < 1 \), the circuit is always stable, and for \( Q > 1 \), the circuit will oscillate if

\[ \omega_0 Q^2 > GB. \]  

(35)

As an example, for \( Q = 2 \) and using the Fairchild IC, PA741 having \( GB = 2n \times 10^9 \) rad/s, the circuit will oscillate at frequencies above 125 kHz.

VI. Experimental Results

The circuit was built using the pA741 OA and satisfactory results were obtained. For example, when \( K = 6 \), \( R = 1 \) k\( \Omega \), and \( C = 0.02 \) pF (i.e., \( Q = 2.45 \)), \( f_o \) (measured) = 19.1 kHz. Using (9) and (32), the actual theoretical value \( f_o \) = 19.2 kHz. That is, there is 0.5 percent error between \( f_o \) (measured) and \( f_o \).

VII. Conclusions

A new active RC low-pass filter was given. The sensitivities with respect to all passive and active circuit components are very low. The frequency limitations of the filter are derived using the one-pole rolloff model of the OA.

References

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**Poles of Maxim**

**Low-Pass Filters**
Fig. 1. Loci of poles of maximally flat sharp-cutoff low-pass filters.
Proportional Bandwidth Filtering
CHARLES R. GBBENE

Abstract—In power spectrum measurement, there are instances in which proportional bandwidth filtering is preferable to constant bandwidth filtering. One such instance is the analysis of sound and vibration from a rotating machine with rotating components. Typically, the power spectrum measurement is the most prominent method of power spectrum estimation. It is unusual for efficient coverage of some frequency range with neither overlap nor gap between filters, the filter center frequencies should be proportionately spaced.

Fig. 1. Example of one-octave filter coverage on a logarithmic frequency scale.

It can be shown that the requirement for no overlap or gap leads to the same proportionality for the cutoff frequencies,

\[ K_i = c f_l. \]

A general problem is determining how to split an arbitrary range of frequencies from \( F_1 \) to \( F_2 \) into \( n \) proportionately spaced filter bands. Beginning with \( f_i = F_1 \), one can write \( h = c F_1, f_2 = c f_1 = c^2 F_1, \) etc. The last (\( n \))th filter is at \( F_2 \), so

\[ F_2 = c^{n} F_1. \]

Using (6) in (5) one can determine the bandwidth proportionality for \( n \) filters spanning \( F_1 \) to \( F_2 \) Hz.

As an example, consider one-third octave filters. An octave will be split into three parts, \( b \) for a one-octave range there will be four filters. Thus, \( c = 2^{1/3} = 1.25992 \). The corresponding \( k = 0.23156 \); the bandwidth of each filter is 23.16 percent of its center frequency.

Another general problem is determining how many filters will be required to span a specified frequency range with a specified percentage bandwidth. Equations (5) and (6) may be solved to yield

\[ N = 1 + \frac{\log \left( \frac{F_2}{F_1} \right)}{2 \log \left( \frac{k + \sqrt{k^2 + 4V}}{2} \right)} \]

Equation (5) may also be written \([1]\) as

\[ k = 2 \sinh (1/2 \log c). \]

The relationship of \( k \) to \( c \) is of special interest. Appropriate manipulation of (1), (2), and (4) leads to the result

\[ k = \frac{c - 1}{c}. \]

For the past several years, constant bandwidth analysis has been the subject of intense research and has become the dominant method of power spectrum estimation. It is unusual to hear of or see a reference to proportional bandwidth analysis, yet there are applications in sound and vibration studies for which constant bandwidth analysis is not ideal. A prime example is the analysis of noise and vibration from a machine with rotating components. Typically, the power spectrum peaks at the fundamental and several harmonics of the rotation frequency. Any instability in the machine speed is manifested as a variation in the frequencies of the fundamental and harmonics. A variation may be described as a certain percentage change of frequency. For example, if the rotation speed is 600 23 r/min, the fundamental is 10 20.05 Hz, and the tenth harmonic is 100 20.5 Hz. The variation is 1 percent at all harmonics. If the variation of the fundamental frequency roughly matches the bandwidth of a constant bandwidth filter system, the variations of the harmonics will be larger than the analysis bandwidth. The result of a constant bandwidth analysis is likely to be misleading as a consequence of the proportionately large excursions in the higher harmonic components. With proportional bandwidth analysis, if the filter bandwidth at the fundamental frequency matches the fundamental frequency variation, the bandwidths at the higher harmonics will match the harmonic frequency variations.

In a set of proportional filters, the basic relationship between the bandwidth \( A_f \) of the \( i \)th filter and its center frequency.

\[ A_f = k f_i. \]