OPTIMALITY CRITERIA AND DUALITY IN NONDIFFERENTIABLE CONTINUOUS PROGRAMMING AND CONTROL

by

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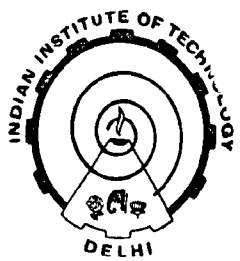

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A B S T R A C T

This thesis entitled, "Optimality Criteria and Duality in Nondifferentiable Continuous Programming and Control" is divided into six chapters, the contents of which are given below.

The first chapter is general introduction which contains preliminaries, review of related work and a summary of the thesis.

The second chapter gives duality results for a class of continuous programming problems under weaker convexity hypotheses. These results generalize various well known results in variational problems, and also give a dynamic analogue of certain recent results on generalized concavity and duality in mathematical programming. Further, a suitable pair of continuous programming problems is constructed and symmetric and self duality results are established under weaker convexity assumptions.

In the third chapter optimality conditions and duality results are obtained for a class of continuous programming problems with a nondifferentiable term in the integrand of the objective function. These results generalize various well-known results in continuous programming problems with differentiable functions, and also provide a dynamic generalization of certain nondifferentiable mathematical

programming problems. This chapter also deals with the optimality criteria and duality results for a class of continuous programming problems with an L_p norm in the integrand of the objective functional. It is also discussed that these results, as in the previous case, can be regarded as a dynamic generalization of those of their static-counterparts in nonlinear programming problems.

The fourth chapter provides a study on symmetric and self-duality for a class of nondifferentiable nonlinear programs which combine differentiable as well as non-differentiable cases appearing in the literature. Various natural extensions are discussed. The study also includes duality and self duality for a pair of symmetric dual non-differentiable continuous programs which are regarded as a dynamic generalization of the earlier problems of this chapter.

The fifth chapter deals with optimality conditions and duality for a class of control problems having a non-differentiable term in the integrand of the objective functional - the nondifferentiability entering in the control vector. These results generalize various well known results in optimal control theory with differentiability and also present a relationship with certain nondifferentiable

mathematical programming problems. Further a unified treatment of optimal control theory and continuous programming are mentioned.

In chapter six, the last chapter of the thesis, optimality conditions and duality results are obtained for a class of continuous nondifferentiable fractional programming problems. These results generalize various well known results in continuous fractional programming and also give a dynamic generalization of certain class of nondifferentiable fractional programming problems.

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