Crystal orientation dependence of the SNR for signal beam amplification in photorefractive BaTiO₃

J. JOSEPH, K. SINGH, P.K.C. PILLAI

In signal beam amplification by two-beam coupling in BaTiO₃ photorefractive crystals, beam fanning in the direction of the amplified signal reduces the signal-to-noise ratio (SNR). The dependence of the SNR and the signal beam gain on the crystal orientation are analysed using a HeNe laser. It is found that orientating the crystal for maximum gain gives poor signal-to-noise ratio. A compromise has to be made between the SNR and high gain for optimum signal amplification.

KEYWORDS: photorefractive crystals, two-beam coupling, signal beam amplification, signal to noise ratio

Introduction

Because of the numerous applications of the photorefractive effect in ferroelectric crystals such as BaTiO₃, (see Ref. 1) there has been a tremendous amount of research in recent years using these crystals. The applications include optical information storage, coherent image amplification, optical phase conjugation, image processing, optical logic operations, and many more. Two-beam coupling is the technique most widely used for coherent image amplification and its applications. A high-gain, low-noise amplified image is the ultimate aim of this technique. Coherent image amplification by two-beam coupling in BaTiO₃ has already been studied in detail, and has considered different parameters such as incident beam ratio, spatial frequency, pump beam intensity and c-axis orientation.

However, to our knowledge, a study of the signal-to-noise ratio (SNR) characteristics of signal beam amplification by two-beam coupling has not been done so far. Also, there is very little literature on the use of a HeNe laser for beam coupling experiments in BaTiO₃. Most of the research work until now has used Ar⁺ lasers. We noticed that, except for the slow response as compared with the green wavelength from an Ar⁺ laser, the 632.8 nm from a HeNe laser is perfectly suitable for all wave-mixing experiments in photorefractive BaTiO₃. We could perform two-wave mixing, four-wave mixing, semi-linear phase conjugation and semi-linear double phase conjugation experiments with good efficiencies. Its easy availability, cheap price and no cooling requirements, give the HeNe laser advantages over the Ar⁺ laser, for the development of novel applications using the photorefractive properties of BaTiO₃.

In signal beam amplification by two-beam coupling, the main source of noise is beam fanning. Beam fanning is generated by amplification of the scattered light, due to two-beam coupling with the incident beam itself. While maximizing the signal beam gain, it can be found that the amplified signal always lies within the angular spread of the fanned beam. This gives rise to very poor signal-to-noise ratios in the amplified signal which could degrade the quality of the image in coherent image amplification experiments. In this paper, we present the results of two-beam coupling experiments on a BaTiO₃ crystal with an emphasis on the dependence of the SNR and the signal beam gain on crystal c-axis orientation. High signal-beam-gain with good SNR are essential for high quality image processing experiments.

Beam amplification by two-beam coupling

Signal beam amplification is the result of interference between two coherent beams inside a non-linear medium and the subsequent energy transfer from one beam to the other. This energy transfer is due to the self-diffraction of the pump beam into the probe beam.
beam and this energy transfer is found to be a maximum when there is a spatial shift of $\frac{\pi}{2}$ between the interference pattern and the induced refractive index pattern. This phase shift of $\pi/2$ gets automatically satisfied for a photorefractive BaTiO$_3$ crystal in the diffraction regime, that is in the absence of an external electric field. Further, the importance of the BaTiO$_3$ crystal lies in the high value of its electro-optic coefficient $r_{32}$ in the electro-optic tensor. In order to exploit this high value both the interfering beams have to be extraordinary polarized and the crystal c-axis has to be tilted away from the grating wave-vector.

The two-beam coupling configuration is described in Fig. 1. Two coherent beams, one a strong pump beam and the other a weak signal beam interfere inside the photorefractive BaTiO$_3$ crystal. The intensity modulation due to interference between the two beams produces a space charge field ($E_{sc}$) modulation inside the crystal, which in turn produces a refractive index modulation due to the Pockels effect. This refractive index modulation is related to the photoinduced space charge field by the following relation:

$$\delta n = \frac{1}{2} n^4 r_{32} E_{sc}$$  (1)

where $n$ is the effective refractive index and $r_{32}$ is the effective electro-optic coefficient.

Following Kukhtarev's theory, the space charge field is given by

$$E_{sc} = E_0 (E_0 + E_0)$$

where $E_0 = k_0 T_k \phi / e$

and $E_0 = e N_0 / k_0 \varepsilon_0$.

Here $N_0$ is the trap density and $\varepsilon$ is the dielectric constant. The grating wave number, $k_0 = 2\pi / \Lambda_g$ with $\Lambda_g = \lambda / 2 \sin (\theta/2)$, where $\lambda$ is the wavelength of the interfering beams and $\theta$ is the angle between the beams.

The exponential gain coefficient $\Gamma$ is related to the refractive index modulation as

$$\Gamma = (4\pi \delta n) / \lambda$$

where $\delta n$ is the maximum value of the index modulation due to the photo refractive effect.

The signal beam gain is defined as,

$$\gamma_o = \frac{I_o}{I_o}$$

with pump beam

$$\gamma_0 = \frac{I_o}{I_o}$$

with no pump

$$= (1 + \beta) e^{\Gamma} / (1 + \beta e^{\Gamma})$$

where $\beta = I_{so} / I_{BO}$ is the intensity ratio of the two incident beams and $I_o$ is the crystal interaction length.

**Experiment and results**

Figure 2 shows the experimental set-up for measurements of signal beam gain, with beam fanning as the noise. The laser used was a 15 mw HeNe laser (Spectra Physics) at a wavelength $\lambda = 632.8$ nm. The BaTiO$_3$ crystal used had dimensions of $5 \times 5 \times 6$ mm$^3$ with the c-axis along the 6 mm edge. The crystal was kept at the centre of a rotation stage. Measurements were taken on a power meter (NRC, Model 830). An aperture of diameter equal to the amplified signal diameter was kept just in front of the power meter.

The pump beam intensity was 0.8 W cm$^{-2}$ at the crystal plane and the input beam intensity ratio ($\beta < 10^{-3}$) was kept small enough to avoid the deleterious effects of pump depletion and high modulation.

Figure 3 shows the variation of the signal beam gain with (external) angle between the writing beams. The continuous curve is obtained from (3) with $N_0 = 2 \times 10^{10}$ cm$^{-3}$ and shows a maximum for $\theta = 27.6^\circ$, corresponding to a fringe spacing $\Lambda_g = 1.32 \mu$m. For the experimental points (dotted curve), the angle between the beams was varied in steps and each time the c-axis orientation was rotated.
to get the maximum gain at that angle. The experimental curve shows a maximum gain for \( \theta = 29^\circ \) corresponding to a fringe spacing \( \Lambda_s = 1.26 \mu \text{m} \).

The signal-to-noise ratio measurements were carried out for an angle of 30° between the beams. The c-axis orientation with respect to the grating wave-vector was varied by rotating the crystal in steps as shown in Fig. 2. Measurements using a power meter were made at each step of the amplified signal power \( P_{s\text{d}} \) (this includes the fanned light as noise) and fanned beam power \( P_{\text{noise}} \) in the direction of the amplified signal beam (with signal beam cut off). From these the signal beam gain (\( \gamma \)) and signal-to-noise ratio (SNR) were calculated as follows:

\[
\gamma = \frac{P_{s\text{d}} - P_{\text{noise}}}{P_{\text{si}}}
\]

where \( P_{\text{si}} \) is the signal beam power with pump beam cut off.

\[
\text{SNR} = \frac{P_{s\text{d}} - P_{\text{noise}}}{P_{\text{noise}}}
\]

Figure 4 shows the variation of gain \( \gamma \) and SNR with crystal orientation. The gain shows a maximum for \( \phi = 72^\circ \) which corresponds to a tilt of \( \sim 19.5^\circ \) between the c-axis and grating wave-vector. The SNR maximum occurs for \( \phi = 40^\circ \) after which the SNR decreases to a minimum (at \( \phi = 78^\circ \)) and shows an increase after this.

From Fig. 4 it is found that orientating the crystal for maximum signal gain gives rise to very poor SNR because of the amplified scattered light in the direction of the amplified signal beam. The crystal orientations for maximum gain and minimum SNR nearly coincide with each other. For high signal-to-noise ratios it is seen that this has to be done at the cost of a reduction in gain. Hence, a compromise has to be made between these two, depending on the application requirements. Crystal orientation \( \phi \) ranging from 30° to 60° could be a good compromise where the high gain is retained for good SNRs.

**Conclusion**

Signal beam amplification has been investigated by two-wave mixing in a photorefractive BaTiO₃ crystal using a HeNe laser. A gain of \( \sim 2500 \) has been obtained for a 30° angle between the beams with a beam intensity ratio of \( > 10^6 \). Studies on the dependence of signal beam gain and SNR on crystal orientation have been carried out. It is found that orientating the crystal for the maximum signal gain results in poor SNR. A compromise has to be made between high gain and SNR. Our results could be useful where high quality image processing experiments are to be performed using two-wave mixing.

**Acknowledgements**

Experimental facilities used in this work were created as a result of funding by the Department of Science and Technology, Government of India and Ministry of Human Resources Development, Government of India. The authors wish to thank Professor M.S. Sodha for his interest and encouragement.

**References**

(1989) 1275–1277
15 Feinberg, J. 'Asymmetric self-defocussing of an optical beam
16 Valley, G.C., 'Competition between forward- and backward-
B4 (1987) 14–19
17 Ewbank, M.D., Neurgaonkar, R.R., Cory, W.K., Feinberg, J.
'Photorefractive properties of strontium barium niobate.' J. Appl
18 Kukhtarev, N.V. et al. 'Holographic storage in electrooptic
crystals. I: Steady state.' Ferroelectrics 22 (1979) 949–960