Channel Capacity of Adaptive Transmission With Maximal Ratio Combining in Correlated Rayleigh Fading

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Abstract—We derive closed-form expressions for the single-user capacity of maximal ratio combining diversity systems taking into account the effect of correlation between the different branches. We consider a Rayleigh fading channel with two kinds of correlation: 1) equal branch signal-to-noise ratios (SNRs) and the same correlation between any pair of branches and 2) unequal branch SNRs and arbitrary correlation between branches such that the eigenvalues of the branch covariance matrix are all distinct. Three adaptive transmission schemes are analyzed: 1) optimal simultaneous power and rate adaptation; 2) optimal rate adaptation with constant transmit power; and 3) channel inversion with fixed rate.

Index Terms—Adaptive transmission, channel capacity, correlated Rayleigh fading, maximal ratio combining (MRC).

I. INTRODUCTION

SHANNON’S landmark paper [1] established the significance of channel capacity as the maximum possible rate at which information can be transmitted over a channel. Thus, the Shannon capacity provides a benchmark against which the spectral efficiency of practical transmission strategies can be compared. The Shannon capacity of fading channels under different assumptions about transmitter and receiver channel knowledge has been examined in [2]–[4] and the references therein. These capacity results can be used to compare the effectiveness of both adaptive and nonadaptive transmission strategies [5], [6] in fading channels against their theoretical maximum performance. In [7], applying the general theory developed in [3], closed-form expressions for the single-user Shannon capacity were derived for a maximal ratio combining (MRC) diversity system under different adaptive transmission techniques, assuming multiple uncorrelated branches with equal average signal-to-noise ratio (SNR). However, independent fading is not always realized in practice. For example, small-size terminals with space antenna diversity may have insufficient antenna spacing to obtain independent fading in each branch. As a result, the maximum theoretical diversity gain cannot be achieved. In addition, the diversity branches in a practical system may have unequal average SNRs due to different noise figures or feedline lengths.

In this paper, we extend the results presented in [7] to obtain closed-form expressions for the single-user capacity of MRC diversity systems taking into account the effect of correlation between the branches. Both the cases of balanced and unbalanced branch SNRs are dealt with. We consider a slow nonselective Rayleigh fading channel with two kinds of correlation between branches:

1) equal branch SNRs and the same correlation between any pair of branches;
2) unequal branch SNRs and arbitrary correlation between branches such that the eigenvalues of the branch covariance matrix are all distinct.

We assume the channel to be block-stationary and ergodic, which implies block-stationarity and ergodicity of the pathstrength or branch-strength distributions. Three adaptive transmission schemes [3], [7] are analyzed: 1) optimal simultaneous power and rate adaptation; (2) optimal rate adaptation with constant transmit power; and (3) channel inversion with fixed rate. The first scheme achieves the ergodic capacity of the system: the maximum achievable average rate under adaptive transmission [3], [8], [9]. However, this average rate is achieved by varying the rate and power relative to the channel conditions, which may not be appropriate for applications requiring a fixed rate. The second scheme also entails variable-rate transmission relative to the channel but is more practical since the transmit power remains constant. The last scheme achieves the outage capacity of the system, defined as the maximum constant transmission rate that can be supported under all channel conditions with some outage probability [8], [9]. When this outage probability is zero, this capacity is also called the zero-outage or delay-limited capacity. The zero-outage capacity is always less than the ergodic capacity since the flexibility of varying the
rate relative to the channel conditions is not possible. However, the constant-rate policy associated with zero-outage capacity is highly desirable for some applications and, therefore, worth some capacity penalty. The analysis assumes that the receiver has perfect knowledge of the branch amplitudes and phases for the diversity combining and that the transmitter has perfect knowledge of the combined SNR and can adapt to it.

The paper is organized as follows. In Section II, we obtain the probability density function (pdf) of the combiner output SNR. Section III presents the derivation of a closed-form expression for the channel capacity in the case of optimal simultaneous power and rate adaptation. The case of optimal rate adaptation with constant transmit power is dealt with in Section IV. Section V discusses the case of channel inversion with fixed rate. Numerical results are given in Section VI. Section VII contains some concluding remarks.

II. PDF OF COMBINER OUTPUT SNR

Consider the coherently received signal with $L$ diversity branches and predetection MRC. Let $\gamma_i$, $i = 1, \ldots, L$, denote the instantaneous SNR of the $i$th diversity branch. Each of the random variables $\sqrt{\gamma_1}, \ldots, \sqrt{\gamma_L}$ has a marginal distribution which is Rayleigh. When MRC is used, the combiner output SNR is given by

$$\gamma = \frac{1}{\log_2(\gamma_1 \cdots \gamma_L)}$$

The statistics of the branch SNRs $\gamma_1, \ldots, \gamma_L$ are generated from Gaussian random variables in the following way. Let

$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_L \end{bmatrix}, \quad Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_L \end{bmatrix}$$

denote two independent and identically distributed (i.i.d.) real-valued Gaussian random vectors with mean zero and covariance matrix $K$, such that

$$\gamma_i = X_i^2 + Y_i^2, \quad i = 1, \ldots, L.$$ (3)

We shall call $K$ the branch covariance matrix. The characteristic function (cf) of the combiner output SNR $\gamma$ in (1) can be expressed as

$$\Psi_\gamma(j\omega) = \mathbb{E}[e^{j\omega\gamma}] = \frac{1}{(\log_2(\gamma_1 \cdots \gamma_L))^{-1}}$$

where $\mathbb{E}[\cdot]$ denotes the expectation and $I_L$ is the $L \times L$ identity matrix.

We consider two cases:

1. The covariance matrix $K$ is a matrix of constant correlations with correlation coefficient $\rho$, where $0 < \rho < 1$, and is given by

$$(K)_{ij} = \begin{cases} \sigma_i^2, & \text{if } i = j \\ \rho \sigma_i \sigma_j, & \text{if } i \neq j, \ i, j = 1, \ldots, L. \end{cases}$$

(5)

2. The eigenvalues of $K$, denoted as $\lambda_1/2, \ldots, \lambda_L/2$, are all distinct.

A. Constant Correlation Case

When $K$ is a matrix of constant correlations, it can be easily shown from (3) and (5) that $\mathbb{E}[\gamma_i] = 2a^2 = \gamma_{av}, \ i = 1, \ldots, L$, and

$$\mathbb{E}[\gamma_i \gamma_j] = \begin{cases} 2\gamma_{av}^2, & \text{if } i = j \\ \gamma_{av}^2(1 + \rho^2), & \text{if } i \neq j, \ i, j = 1, \ldots, L. \end{cases}$$

(6)

From (4) and (5), the cf of $\gamma$ is given by

$$\Psi_\gamma(j\omega) = \frac{1}{(1 - j(1 - \rho)\gamma_{av}\omega)^{L-1}(1 - j(1 + \rho)\gamma_{av}\omega)^{L-1}}.$$ (7)

Denoting

$$a = \frac{1}{\gamma_{av}(1 + [L - 1]\rho)}, \quad b = \frac{1}{\gamma_{av}(1 - \rho)}$$

and using partial fractions, (7) can be rewritten as

$$\Psi_\gamma(j\omega) = \frac{(-1)^L ab^{L-1}}{(j\omega - a)(j\omega - b)^{L-1}} = (-1)^L ab^{L-1}$$

$$\times \frac{A_{11}}{(j\omega - a)} + \sum_{k=1}^{L-1} \frac{A_{2k}}{(j\omega - b)^k}$$

(9a)

where

$$A_{11} = \frac{1}{(a - b)^{L-1}}$$

$$A_{2k} = \frac{1}{(L - k - 1)!} \left[ \frac{a^{L-k-1}}{d(j\omega - b)(j\omega - a)^{L-k-1}} \right]_{j\omega = b}$$

$$= (-1)^{L-k-1} \frac{e^{b(a-b)}}{(a-b)^{L-k}(k-1)!}, \quad k = 1, \ldots, L - 1.$$ (9b)

Using the result

$$\frac{1}{(k-1)!} \int_0^\infty e^{\omega \gamma} \gamma^{k-1} d\gamma = \frac{1}{(c - j\omega)^k}, \quad c > 0, \quad k = 1, 2, 3, \ldots$$

(10)

in (9), we obtain the following expression for the pdf of $\gamma$:

$$f_\gamma(v) = ab^{L-1} \left[ e^{-av} \sum_{k=1}^{L-1} \frac{v^{k-1}}{(a-b)^{L-k}(k-1)!} \right], \quad v \geq 0.$$ (11)

In addition, the indefinite integration result

$$\int v^{k-1} e^{-av} dv = \frac{1}{(c - j\omega)^k} \sum_{k=1}^{L-1} \frac{v^{k-1}}{a^{L-k}((a-b)^{L-k}) \prod_{n=0}^{k-1} (an)!}, \quad c > 0, \quad k = 1, 2, 3, \ldots$$

(12)

the primitive of $f_\gamma(\cdot)$, which we denote as $F_\gamma(\cdot)$, can be expressed as

$$F_\gamma(v) = \int f_\gamma(v) dv$$

$$= ab^{L-1} \left[ -e^{-av} \frac{1}{a(b-a)^{L-1}} \sum_{k=1}^{L-1} \frac{(bv)^{k-1}}{a^{L-k}((a-b)^{L-k}) \prod_{n=0}^{k-1} (an)!} \right].$$ (13)
Note from (13) that \( F_{\gamma}(0) = -1 \). Let \( F_\gamma(\cdot) \) denote the cumulative distribution function (cdf) of \( \gamma \). This cdf can be expressed in terms of the primitive as

\[
F_\gamma(v) = F_\gamma(v) - F_\gamma(0) = 1 + F_\gamma(v).
\]

### B. Distinct Eigenvalues Case

When \( K \) has distinct eigenvalues \( \lambda_1/2, \ldots, \lambda_L/2 \), the pdf of \( \gamma \) is given by [10]

\[
f_\gamma(v) = \frac{1}{\prod_{j=1}^{L} \lambda_j} \prod_{j=1}^{L} \prod_{k \neq j} \left( \frac{1}{\lambda_k} - \frac{1}{\lambda_j} \right) \exp \left( -\frac{v}{\lambda_j} \right), \quad v \geq 0
\]

and its primitive \( F_\gamma(\cdot) \) is

\[
F_\gamma(v) = -\frac{1}{\prod_{j=1}^{L} \lambda_j} \sum_{j=1}^{L} \prod_{k \neq j} \left( \frac{1}{\lambda_k} - \frac{1}{\lambda_j} \right) \exp \left( -\frac{v}{\lambda_j} \right).
\]

### III. Optimal Simultaneous Power and Rate Adaptation

Under this condition, which we shall refer to as \( \text{opra} \), the channel capacity \( C_{\text{opra}} \) (in bits/second) of a fading channel with received SNR pdf \( f_\gamma(\cdot) \) is given by [3], [7]

\[
C_{\text{opra}} = B \ln 2 \int_{\gamma_0}^{\infty} \ln \left( \frac{v}{\gamma_0} \right) f_\gamma(v) \, dv
\]

where \( B \) (in hertz) is the channel bandwidth and \( \gamma_0 \) is the optimal cutoff SNR satisfying

\[
\int_{\gamma_0}^{\infty} \left( \frac{1}{\gamma_0} - \frac{1}{v} \right) f_\gamma(v) \, dv = 1.
\]

Using integration by parts, we can rewrite (16) as

\[
C_{\text{opra}} \ln 2 = - \int_{\gamma_0}^{\infty} \frac{1}{v} F_\gamma(v) \, dv.
\]

### A. Constant Correlation Case

When \( K \) is a matrix of constant correlations, substitution of (13) in (18) gives

\[
C_{\text{opra}} \ln 2 = \frac{b^{\frac{L}{2}} \beta^{\frac{L}{2}}}{(b-a)^{\frac{L}{2}} - 1} \int_{\gamma_0}^{\infty} \frac{e^{-a \gamma}}{v} \, dv - \sum_{k=1}^{L} \frac{b^{L-k}}{(b-a)^{L-k}} \int_{\gamma_0}^{\infty} \frac{e^{-a \gamma}}{v} \, dv + \sum_{n=0}^{L} \frac{b^v}{n!} \int_{\gamma_0}^{\infty} v^{n-1} e^{-b \gamma} \, dv.
\]

Let us denote the exponential integral of order one by

\[
E_1(c) = \int_{1}^{\infty} \frac{e^{-c \gamma}}{\gamma} \, d\gamma, \quad c \geq 0
\]

and the Poisson distribution by

\[
P_k(c) = e^{-c} \sum_{n=0}^{k} \frac{c^n}{n!}
\]

Observe from (21) that \( P_k(0) = 1 \).

Substituting (12), (20), and (21) in (19), the following closed-form expression for the capacity per unit bandwidth (in bits/second/Hz) is obtained:

\[
C_{\text{opra}} \ln 2 = \frac{1}{\ln 2} \left[ E_1(a \gamma_0) \left( \frac{b}{b-a} \right) - E_1(b \gamma_0) \left( \frac{b}{b-a} \right) - 1 \right] + \sum_{n=0}^{L-1} \frac{P_n(b \gamma_0)}{n} \left( \frac{b}{b-a} \right)^{-n-1} - 1
\]

\[
= \frac{1}{\ln 2} \left[ E_1 \left( \frac{\gamma_0}{\gamma_0 (1 + [L-1] \rho)} \right) \left( 1 + [L-1] \rho \right) - E_1 \left( \frac{\gamma_0}{\gamma_0 (1 + \rho)} \right) \left( 1 + \rho \right) - 1 \right] + \sum_{n=0}^{L-1} \frac{P_n(b \gamma_0)}{n} \left( \frac{b}{b-a} \right)^{-n-1} - 1.
\]

To obtain the optimal cutoff SNR \( \gamma_0 \) in (22), we need to solve for \( \gamma_0 \) in (17), which can be rewritten as

\[
\frac{1}{\gamma_0} F_\gamma(\gamma_0) - \int_{\gamma_0}^{\infty} \frac{1}{v} f_\gamma(v) \, dv = 1
\]

where \( F_\gamma(\cdot) \) is given by (13). It can be shown from (11) and (12) that

\[
\int_{\gamma_0}^{\infty} \frac{1}{v} f_\gamma(v) \, dv = \frac{ab^{L-1}}{(b-a)^L} [E_1(a \gamma_0) - E_1(b \gamma_0)]
\]

\[
= \sum_{k=1}^{L-1} \frac{b^{L-k}}{(b-a)^L-k} P_{k-1}(b \gamma_0).
\]

Using (13) and (24), we get from (23) the equation

\[
\frac{ab^{L-1}}{(b-a)^L} \left[ \frac{e^{-a \gamma_0} - e^{-b \gamma_0}}{\gamma_0} - E_1(a \gamma_0) + E_1(b \gamma_0) \right]
\]

\[
= \sum_{k=1}^{L-1} \frac{b^{L-k}}{(b-a)^L-k} \left( P_k(b \gamma_0) - P_k(b \gamma_0) \right).
\]

where \( a \) and \( b \) are given by (8). For specific values of \( a \) and \( b \), (25) has to be solved numerically to obtain \( \gamma_0 \).

Since the transmission is suspended when \( \gamma < \gamma_0 \), there is an outage probability which is given by

\[
P_{\text{out}} = 1 - \int_{\gamma_0}^{\infty} f_\gamma(v) \, dv = 1 + F_\gamma(\gamma_0)
\]

\[
= 1 - \left( \frac{b}{b-a} \right)^L e^{-a \gamma_0} + \sum_{k=1}^{L-1} \left( \frac{b}{b-a} \right)^L P_k(b \gamma_0).
\]
In comparison, when $\rho = 0$, the branch SNRs $\gamma_1, \ldots, \gamma_L$ are i.i.d., and we get [7]

$$
\frac{C_{\text{opt}}}{B} \bigg|_{\rho=0} = \frac{1}{\ln 2} \left[ E_1 \left( \frac{\gamma_0}{\gamma_{0v}} \right) + \sum_{n=1}^{L-1} \frac{1}{n} P_n \left( \frac{\gamma_0}{\gamma_{0v}} \right) \right]
$$

(27a)

where the cutoff SNR $\gamma_0$ satisfies the equation

$$
\frac{1}{\gamma_0} P_L \left( \frac{\gamma_0}{\gamma_{0v}} \right) - \frac{1}{\gamma_{0v}(L-1)} P_{L-1} \left( \frac{\gamma_0}{\gamma_{0v}} \right) = 1
$$

(27b)

and the outage probability is given by

$$
P_{\text{out}} \bigg|_{\rho=0} = 1 - P_L \left( \frac{\gamma_0}{\gamma_{0v}} \right).
$$

(27c)

These expressions are quite different from what we obtained for the correlated case because when $\rho = 0$, the pdf of $\gamma$ is

$$
f_\gamma(v) \bigg|_{\rho=0} = \frac{1}{\gamma_{0v}(L-1)} v^{L-1} e^{-v} I_v \gamma_{0v}, \quad v \geq 0
$$

(28)

and cannot be obtained by directly substituting $a = b = 1/\gamma_{0v}$ in (11).

B. Distinct Eigenvalues Case

Substituting (15) into (18) and using (20) we obtain

$$
\frac{C_{\text{opt}}}{B} = \frac{1}{\ln 2} \left[ \frac{1}{\prod_{j=1}^L \lambda_j} \sum_{j=1}^L \lambda_j E_1 \left( \frac{\lambda_j}{\gamma_0} \right) \right].
$$

(29)

Further, substituting (14) into (17), we find that $\gamma_0$ must satisfy

$$
\frac{1}{\prod_{j=1}^L \lambda_j} \sum_{j=1}^L \lambda_j \exp \left( -\frac{\lambda_j}{\gamma_0} \right) - E_1 \left( \frac{\lambda_j}{\gamma_0} \right) = 1.
$$

(30)

Using (15), the outage probability is given by

$$
P_{\text{out}} = 1 + F_{\gamma}(\gamma_0)
$$

$$
= 1 - \frac{1}{\prod_{j=1}^L \lambda_j} \sum_{j=1}^L \lambda_j \exp \left( -\frac{\lambda_j}{\gamma_0} \right).
$$

(31)

IV. OPTIMAL RATE ADAPTATION WITH CONSTANT TRANSMIT POWER

In this case, which we shall refer to as $\rho = 0$, the channel capacity is given by [2], [3], [7], [11]

$$
\frac{C_{\text{opt}}}{B} = \frac{B}{\ln 2} \int_0^\infty \ln (1 + v) f_\gamma(v) \, dv.
$$

(32)

Applying integration by parts, we obtain

$$
\frac{C_{\text{opt}}}{B} \ln 2 = - \int_0^\infty \frac{1}{(1 + v)^2} F_\gamma(v) \, dv.
$$

(33)

A. Constant Correlation Case

In this case, a change of variable $x = 1 + v$ on the right-hand side of (33) yields

$$
\frac{C_{\text{opt}}}{B} \ln 2 = \frac{b L - 1}{(b - a)^{L-1}} E_1(a) - a e^b \sum_{k=1}^{L-1} \frac{b^{L-k-1}}{(b - a)^{L-k}}
$$

$$
\times \sum_{n=0}^{k-1} \frac{b^n}{n!} \int_1^\infty (x - 1)^n e^{-bx} \, dx.
$$

(34)

Using the binomial expansion of $(x - 1)^n$ along with (12), (20), and (21), we get from (34) the following expression for the capacity per unit bandwidth:

$$
\frac{C_{\text{opt}}}{B} = \frac{1}{\ln 2} \left[ \left( \frac{b}{b - a} \right)^{L-1} e^a E_1(a) - \frac{a}{b} e^b \sum_{k=1}^{L-1} \frac{b^{L-k-1}}{(b - a)^{L-k}}
$$

$$
\times \left\{ P_k(-b) E_1(b) + \sum_{m=1}^{k-1} \frac{1}{m} P_m(b) P_{k-m}(-b) \right\} \right].
$$

(35)

When $\rho = 0$, the channel capacity is given by [7], [11]

$$
\frac{C_{\text{opt}}}{B} \bigg|_{\rho=0} = \frac{1}{\ln 2} \left[ P_L \left( -\frac{1}{\gamma_{0v}} \right) E_1 \left( \frac{1}{\gamma_{0v}} \right)
$$

$$
+ \sum_{m=1}^{L-1} \frac{1}{m} P_m \left( \frac{1}{\gamma_{0v}} \right) P_{L-m} \left( -\frac{1}{\gamma_{0v}} \right) \right].
$$

(36)

which is different from (35) in structure.

B. Distinct Eigenvalues Case

Substituting (15) in (33) we get

$$
\frac{C_{\text{opt}}}{B} = \frac{1}{\ln 2} \left[ \sum_{j=1}^L \lambda_j \exp \left( \frac{\lambda_j}{\gamma_{0v}} \right) E_1 \left( \frac{1}{\gamma_{0v}} \right)
$$

$$
\times \left\{ P_k \left( -\frac{1}{\gamma_{0v}} \right) E_1 \left( \frac{1}{\gamma_{0v}} \right)
$$

$$
+ \sum_{m=1}^{k-1} \frac{1}{m} P_m \left( \frac{1}{\gamma_{0v}} \right) P_{k-m} \left( -\frac{1}{\gamma_{0v}} \right) \right\} \right].
$$

(37)
V. CHANNEL INVERSION WITH FIXED RATE

In this case, there are two schemes: truncated channel inversion with fixed rate, which we shall refer to as trifr, and channel inversion with fixed rate without truncation, which we shall refer to as cifr. Note that cifr is also called total channel inversion with fixed rate. In trifr, the transmitter inverts the channel fading by adapting its power level to maintain a constant SNR at the receiver, as long as the fade depth is above a cutoff \( \gamma_0 \). If we set \( \gamma_0 = 0 \), the scheme is cifr.

A. Constant Correlation Case

The channel capacity per unit bandwidth with the truncation scheme is given by

\[
\frac{C_{\text{trfr}}}{B} = \frac{1}{\ln 2} \ln \left( 1 + \frac{1}{\int_{\gamma_0}^{\infty} \frac{1}{v} f_\gamma(v) \, dv} \right) (1 - P_{\text{out}}) \tag{38a}
\]

where, from (24) and (8)

\[
\int_{\gamma_0}^{\infty} \frac{1}{v} f_\gamma(v) \, dv = \frac{1}{\gamma_0 \sqrt{\pi}} \left( 1 + \frac{L - 1}{\rho} \right)^{-\frac{1}{2}} \times \left( \frac{1 + \frac{L - 1}{\rho}}{L \rho} \right)^{L - 1} \left[ E_1 \left( \frac{\gamma_0}{\gamma_0 + (L - 1)\rho} \right) - E_1 \left( \frac{\gamma_0}{\gamma_0 (1 - \rho)} \right) \right] - \frac{L - 1}{\rho} \sum_{k=2}^{L - 1} \frac{1}{(k - 1)} \left( \frac{\gamma_0}{\gamma_0 (1 - \rho)} \right)^{L - k} \tag{38b}
\]

and, from (26) and (8)

\[
1 - P_{\text{out}} = \left( 1 + \frac{L - 1}{\rho} \right)^{-\frac{1}{2}} \gamma_0 \sqrt{\pi} \left( 1 + \frac{L - 1}{\rho} \right)^{-\frac{1}{2}} \left( \frac{1 - \rho}{1 + \frac{L - 1}{\rho}} \right)^{L - 1} \sum_{k=2}^{L - 1} \frac{1}{(k - 1)} \left( \frac{\gamma_0}{\gamma_0 (1 - \rho)} \right)^{L - k} \tag{38c}
\]

The cutoff level \( \gamma_0 \) can be chosen either to achieve a specific outage probability \( P_{\text{out}} \), or to maximize (38a). If it is chosen to achieve a given \( P_{\text{out}} \), then (38a) defines the outage capacity corresponding to this \( P_{\text{out}} \).

If we set \( \gamma_0 = 0 \) in (38), we get the capacity for channel inversion with fixed rate and without truncation, i.e., the zero-outage capacity. In this case, \( P_{\text{out}} = 0 \). To obtain an expression for \( \int_{\gamma_0}^{\infty} \frac{1}{v} f_\gamma(v) \, dv \), we use the expansion

\[
E_1(c) = -E - \ln c - \sum_{k=1}^{\infty} \frac{(-c)^k}{k!} \quad c \geq 0 \tag{39}
\]

(Where \( E = 0.577215665 \) is the Euler constant) in (38b), which results in

\[
\lim_{\gamma_0 \to 0} \left[ E_1 \left( \frac{\gamma_0}{\gamma_0 (1 + [L - 1] \rho)} \right) - E_1 \left( \frac{\gamma_0}{\gamma_0 (1 - \rho)} \right) \right] = \ln \left( \frac{1 + [L - 1] \rho}{1 - \rho} \right). \tag{40}
\]

Therefore, for channel inversion with fixed rate and without truncation, the channel capacity per unit bandwidth is given by

\[
\frac{C_{\text{cifr}}}{B} = \frac{1}{\ln 2} \ln \left( 1 + \frac{1}{\int_{\gamma_0}^{\infty} \frac{1}{v} f_\gamma(v) \, dv} \right) \tag{41a}
\]

where, from (38b) and (40)

\[
\int_{\gamma_0}^{\infty} \frac{1}{v} f_\gamma(v) \, dv = \frac{1}{\gamma_0 (1 + [L - 1] \rho)} \times \left( \frac{1 + [L - 1] \rho}{L \rho} \right)^{L - 1} \ln \left( \frac{1 + [L - 1] \rho}{1 - \rho} \right) - \frac{L - 1}{\rho} \sum_{k=2}^{L - 1} \frac{1}{(k - 1)} \left( \frac{1 + [L - 1] \rho}{L \rho} \right)^{L - k} \frac{1}{(k - 1)} \right). \tag{41b}
\]

On the other hand, when \( \rho = 0 \), the capacities are given by

\[
\frac{C_{\text{trfr}}}{B} = \frac{1}{\ln 2} \ln \left( 1 + \frac{\gamma_0 (L - 1)}{P_{L-1} \left( \frac{2 \rho}{\gamma_0} \right)} \right) \quad P_L \left( \frac{\gamma_0}{\gamma_0} \right) \tag{42}
\]

B. Distinct Eigenvalues Case

With the truncation scheme, the channel capacity per unit bandwidth \( C_{\text{trfr}}/B \) is given by (38a), where, from (14), we have

\[
\int_{\gamma_0}^{\infty} \frac{1}{v} f_\gamma(v) \, dv = \frac{1}{\ln 2} \sum_{j=1}^{L} \frac{1}{\prod_{k=1}^{L} \lambda_j} \frac{1}{\prod_{k=1}^{L} \left( \frac{1}{\lambda_k} - \frac{1}{\lambda_j} \right)} \tag{43}
\]

and, from (31)

\[
1 - P_{\text{out}} = \frac{1}{L} \sum_{k=1}^{L} \lambda_k \exp \left( -\frac{\gamma_0}{\lambda_k} \right). \tag{44}
\]

As in the case of constant correlations, \( \gamma_0 \) can be chosen either to achieve a specific \( P_{\text{out}} \), or to maximize (38a).

To obtain the capacity for channel inversion with fixed rate and without truncation, we put \( \gamma_0 = 0 \). By taking the limit of (43) as \( \gamma_0 \) approaches zero, we get, using (39)

\[
\int_{\gamma_0=0}^{\infty} \frac{1}{v} f_\gamma(v) \, dv = \lim_{\gamma_0 \to 0} \frac{1}{\prod_{k=1}^{L} \lambda_j} \sum_{j=1}^{L} \frac{1}{\prod_{k=1}^{L} \left( \frac{1}{\lambda_k} - \frac{1}{\lambda_j} \right)} \tag{45}
\]
Using the fact that
\[
\frac{1}{\prod \lambda_j} \sum_{j=1}^{L} \frac{1}{L-1} \prod_{k=1}^{L} \left( \frac{1}{\lambda_k} - \frac{1}{\lambda_j} \right) = 0
\]
equation (45) simplifies to
\[
\int_{0}^{\infty} f_{\gamma}(\nu) d\nu = \frac{1}{\prod \lambda_j} \sum_{j=1}^{L} \frac{L}{L-1} \prod_{k=1}^{L} \left( \frac{1}{\lambda_k} - \frac{1}{\lambda_j} \right). \tag{46}
\]
Therefore, combining (41a) and (46), we get
\[
\frac{C_{\text{eff}}}{B} = \frac{1}{\ln 2} \ln \left( 1 + \frac{\prod_{j=1}^{L} \lambda_j}{\sum_{j=1}^{L} \prod_{k=1}^{L} \left( \frac{1}{\lambda_k} - \frac{1}{\lambda_j} \right)} \right). \tag{47}
\]
Fig. 5. Comparison of capacity per unit bandwidth (in bits/second/hertz) versus correlation coefficient $\rho$ for different schemes when diversity order $L = 4$, average received SNR per branch $\gamma_{av} = 10$ dB, outage probability $P_{out}$ for $tifr = 0.01$.

VI. NUMERICAL RESULTS

Plots of the channel capacity per unit bandwidth are shown in Figs. 1–5 for correlated fading with constant correlations. We find from the plots that the capacity for all schemes decreases with increase of the correlation coefficient $\rho$. Moreover, the capacity increases with increase of the diversity order $L$ and increase of the average received SNR per branch $E[\gamma_k] = \gamma_{av}$ as expected. However, in the case of opr\textit{a}, the effect of increasing $\rho$ on the capacity $C_{opr\textit{a}}/B$ becomes more prominent with decrease of $\gamma_{av}$ (as seen in Fig. 1(a)) or increase of $L$ (as seen in Fig. 1(b)). It is also to be noted that the decrease in capacity with increase in $\rho$ is much sharper in the case of opr\textit{a} as compared to the other schemes. In the case of tifr, the cutoff SNR $\gamma_0$ which maximizes the capacity decreases with increase of $\rho$, as seen in Fig. 4. A comparison of the plots for the different schemes in Fig. 5 shows that for the same channel bandwidth $B$, $C_{opr\textit{a}} \geq C_{ora} \geq C_{tifr} \geq C_{cifr}$, as expected. The gap between $C_{opr\textit{a}}$ and $C_{ora}$ is lower than that between $C_{ora}$ and $C_{tifr}$. For very large values of $\rho$, however, as $\rho$ decreases, the gap between $C_{opr\textit{a}}$ and $C_{ora}$ increases rapidly while that between $C_{ora}$ and $C_{tifr}$ shows a gradual decrease. The gap between $C_{tifr}$ and $C_{cifr}$ depends on the outage probability $P_{out}$ chosen to compute the cutoff rate $\gamma_0$ for $C_{tifr}$ ($P_{out}$ for $tifr = 0.01$ in Fig. 5). Note the large penalty between $C_{cifr}$ and $C_{opr\textit{a}}$, the capacity penalty between ergodic capacity and zero-outage capacity. This indicates the capacity penalty that arises from a requirement for constant-rate transmission in all channel states rather than adapting the transmission rate to the channel conditions.

In Figs. 6–8, we present plots for uncorrelated fading with imbalanced diversity branches, which is a special situation of the case when the branch covariance matrix $K$ has distinct eigenvalues. Here, if $\lambda_1/2, \ldots, \lambda_L/2$ are the distinct eigenvalues of $K$, then the average received SNR of the $k$th branch is given by

$$\gamma_{avk} = E[\gamma_k] = \lambda_k, \quad k = 1, \ldots, L.$$  

We assume that the SNR imbalance is exponential, that is

$$\gamma_{avk} = \gamma_{av} \exp(-(k-1)\delta), \quad \delta > 0, \quad k = 2, \ldots, L$$

where $\delta$ is the SNR imbalance parameter. The average received SNR per branch is given by

$$\gamma_{av} = \frac{1}{L} \sum_{k=1}^{L} \gamma_{avk}.$$  

In Fig. 6, $C_{opr\textit{a}}/B$ is plotted against $\gamma_{av}$ for $\delta = 0.1, 0.5, 0.9$ when $L = 4$. We find that the capacity increases with increase of $\gamma_{av}$, and decreases slightly with increase of the imbalance parameter $\delta$. In Fig. 7, we plot the outage probability $P_{out}$ versus $\gamma_{av}$ when $L = 4$ for opr\textit{a} and tifr. From this figure, we see that the SNR imbalance parameter has a greater influence on $P_{out}$ for opr\textit{a} than it has on tifr. In addition, the decrease in outage probability with increase in $\gamma_{av}$ is faster for opr\textit{a} than for tifr. Fig. 8 shows the capacity with uncorrelated two-branch and four-branch diversity and power imbalance for the different adaptive transmission techniques. We see from this figure that opr\textit{a} and ora suffer similar capacity penalties due to a decrease in $\gamma_{av}$, Tifr and cifr also suffer capacity penalties with decreasing $\gamma_{av}$. However, the disparity between the two schemes reduces as $\gamma_{av}$ is increased. It is also seen
Fig. 6. Variation of $C_{\text{four}}/B$ (in bits/seconds/Hz) with average received SNR per branch $\gamma_{\text{SNR}}$ when diversity order $L = 4$ for uncorrelated fading and exponential SNR imbalance.

Fig. 7. Variation of $P_{\text{out}}$ with average received SNR per branch $\gamma_{\text{SNR}}$ when diversity order $L = 4$ for uncorrelated fading and exponential SNR imbalance. The top set of lines are for tiff; the bottom set of lines are for oprn.
that increasing the number of branches tends to decrease the disparity in capacity between the different schemes.

VII. CONCLUSION

We have examined the single-user capacity of MRC in a correlated Rayleigh fading environment. We considered three adaptive transmission schemes over two different correlation scenarios. In all cases, we derived closed-form expressions in terms of well-known tabulated functions (the exponential integral of order one and the Poisson distribution). The mathematical formalism has been illustrated by several numerical examples studying the effect of correlation and comparing the capacity under different adaptive transmission strategies.

REFERENCES


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