Error Probability of Binary NFSK and DPSK with Postdetection Combining over Correlated Rician Channels

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Abstract—For binary noncoherent orthogonal frequency-shift keying and binary differential phase-shift keying over slow nonselective Rician fading channels having arbitrarily correlated branches and unequal branch powers, a closed-form expression for the symbol-error probability in the case of postdetection equal-gain combining is obtained directly from the characteristic function of the decision variable.

Index Terms—Binary DPSK, binary noncoherent orthogonal FSK, correlated Rician fading, postdetection EGC, symbol-error probability.

I. INTRODUCTION

The symbol-error probability (SEP) for binary noncoherent modulation using diversity combining in Rician fading was analyzed in the past for independent diversity branches [1]-[3]. A formula for the SEP in case of dual diversity for binary noncoherent orthogonal frequency-shift keying (BNFSK) with postdetection equal-gain combining (EGC) over the correlated Rician channel was derived in [4]. Analysis for noncoherent modulation over the correlated Rician channel with postdetection combining was carried out in [5], where integral expressions for the SEP were presented. In this letter, we derive a closed-form expression (which does not contain derivatives, integrals, and special functions) for the SEP of BNFSK and binary differential phase-shift keying (BDPSK) over slow nonselective Rician fading channels with arbitrarily correlated branches and unequal branch powers using postdetection EGC. The method utilizes the characteristic function (c.f.) of the decision variable, and is based on the technique presented in [6].

The letter is organized as follows. In Section II, we obtain the c.f. of the decision variable for BNFSK and BDPSK from which a closed-form expression for the SEP is derived. Numerical results are presented in Section III.

II. ERROR ANALYSIS

Consider a diversity reception system over a flat Rician fading channel with $L$ correlated branches. The sampled signal received over the $i$th diversity branch in a symbol interval of duration $T_s$ can be represented as

$$r_i(t) = \text{Re} \left\{ (a_i e^{-j\psi_i}) s(t) + n_i(t) \right\} e^{j2\pi f_c t},$$

where $s(t)$ is the complex baseband information-bearing signal with average symbol energy $2E_s$, $a_i$ is the random magnitude and $(\psi_i)$, the random phase of the $i$th diversity branch gain, $f_c$ is the carrier frequency, and $n_i(t)$ representing the additive noise is a zero-mean complex white Gaussian random process with two-sided power spectral density $2N_0$. The noise processes $\{n_i(t)\}$ are assumed to be independent but the random complex channel gains $\{a_i e^{-j\psi_i}\}$ are correlated. In addition, each $n_i(t)$ is independent of the gains $c_{i1} e^{-j\psi_1}, \ldots, c_{iL} e^{-j\psi_L}$ for $i = 1, \ldots, L$.

Define the complex channel gain vector $g$ as

$$g = [\alpha_1 e^{-j\psi_1}, \ldots, \alpha_L e^{-j\psi_L}]^T = X_c + jX_s,$$

where the in-phase component $X_c$, and the quadrature component $X_s$, are real Gaussian $L\times 1$ random vectors with mean vectors $\mu_c$ and $\mu_s$, and cross-covariance matrices $K_{cc}$ and $K_{cs}$, and cross-covariance matrix $K_{cs}$, $E\{X_c X_s^T\} = K_c = \mu_c \mu_s^T + P_c P_s^T$, such that $K_{cc}$ and $K_{cs}$ have the same diagonal elements, and all diagonal elements of $K_{cs}$ are zero, that is

$$(K_{cc})_{ii} = (K_{ss})_{ii}, \quad (K_{cs})_{ii} = 0, \quad i = 1, \ldots, L.$$ 

Let the noncentral $\chi^2(2)$ random vector $(i)$ be defined as

$$\beta = \begin{bmatrix} \beta_1, & \ldots, & \beta_L \end{bmatrix}^T = [\alpha_1^2, \ldots, \alpha_L^2]^T.$$ 

By applying the approach of [7] on the joint distribution of $X_c$ and $X_s$, we can express the c.f. of $(i)$ as

$$\Psi_i(i) = \exp \left\{ E_{\frac{1}{2}} \text{diag}(\mu_i, \mu_i) K_c (I_{2L} - 2\text{diag}(\alpha_i, \alpha_i) K_c)^{-1} \right\}$$

$$\left\{ \det (I_{2L} - 2\text{diag}(\mu_i, \mu_i) K_c) \right\}^{1/2}$$

where

$$K \triangleq \begin{bmatrix} K_{cc} & K_{cs} \\ K_{sc}^T & K_s \end{bmatrix}, \quad \beta = \begin{bmatrix} \mu_c \\ \mu_s \end{bmatrix}$$

and $I_{2L}$ denotes the $2L \times 2L$ identity matrix. To analyze the error performance, we need the c.f. of $P_{i}, K_{i} = J_{2L} \text{diag}(\lambda_1, \ldots, \lambda_L)$ is the diagonal matrix of eigenvalues $A_i$, $\ldots, \sqrt{L}$ of $I_{2L}$ and $E_{\frac{1}{2}}$ is a $2L \times 2L$ orthogonal matrix of eigenvectors of $K_s$. Denoting
For noncoherent detection of binary orthogonal FSK signals, the complex baseband information-bearing signal \( s(t) \) for symbol \( i \) over a symbol interval of length \( T_s \) is given by

\[
s(t) = \sqrt{2E_s / T_s} e^{j\phi_i} \quad i = 0, 1.
\]

The received signal \( r_k(t) \) in the \( k \)th branch is passed through two narrow-band bandpass filters (BPFs), one centered at \( f_c + A_f / \omega \) and the other at \( f_c + A_f / i \). Let \( u_{ki} \) denote the square of the output envelope for the BPF centered at \( f_c + A_f / i \), \( i = 0, 1 \). The decision variable \( D \) resulting from postdetection EGC can be expressed as

\[
D = \sum_{k=1}^{L} u_{k} - \sum_{k=1}^{L} u_{k0}.
\]

Let \( \eta \) denote the instantaneous signal-to-noise ratio (SNR) at the combiner’s output, given by

\[
\eta = \frac{\sum_{k=1}^{L} \beta_{ki}}{\sum_{k=1}^{L} \beta_{k0}}.
\]

where \( \beta_{ki} = \sqrt{2E_s \Delta f_k + N_{ki}} \) is the instantaneous SNR at branch \( k \). When symbol 1 is transmitted, we have

\[
u_{ki} = \sqrt{2E_s \Delta f_k + N_{ki}}; \quad \nu_{k0} = \sqrt{N_{k0}}.
\]

where \( N_{ki} \) and \( N_{k0} \) are i.i.d. complex circular Gaussian random variables with mean zero and \( E[|\mathbf{W}_{ki}|^2] = 4E_s N_0 \) for \( i = 0, 1 \) [8]. For a given output SNR \( \eta \), the conditional c.f. of \( D \) is expressed in a form similar to that in [6] as

\[
\Psi_D (\nu | \eta) = \exp \left\{ \frac{4 ju E_s N_0 \left( 1 - \frac{i}{\eta} \right) \eta}{1 + AJLO \eta T_s (1 - \frac{i}{\eta})} \right\}.
\]

In the case of BDPSK, the complex information-bearing signal over the \( i \)th symbol interval \( (i - 1)T_s < t < iT_s \) is given by

\[
s(t) = \sqrt{2E_s / T_s} e^{j\phi_i} e^{j\theta_i}, \quad \text{and the Mi message symbol is encoded into the phase difference } \theta_i - \theta_{i-1} = 0 \text{ correspond to the symbol 1, and } 81 - 81 - 1 = n \text{ correspond to the symbol 0. The decision variable is given by [6]}
\]

\[
D = 2 \text{Re} \left\{ \sum_{j=1}^{L} v_j(l) v_j^*(l - 1) \right\}
\]

where

\[
v_j(l) = 2E_s g_k e^{j\phi_l} + N_j(l)
\]

is the \( k \)th branch sample for the \( i \)th symbol, and \( N_j(l) \) is a white complex circular Gaussian sequence with mean zero and \( E[|\mathbf{W}_{j}|^2] = 4E_s N_0 \) [8]. When symbol 1 is transmitted, the conditional c.f. \( \psi^{(i)}(\eta | \theta) \) has the same form as (2) with \( E_s / N_0 \) replaced by \( 2E_s / N_0 \) [9].

By averaging \( \psi^{(i)}(\eta | \theta) \) over the p.d.f. of \( \eta \), and using (1), the c.f. of \( D \) becomes

\[
\Psi_D (\nu | \eta) = \exp \left\{ \frac{4 ju E_s N_0 \left( 1 - \frac{1}{\eta} \right) \eta}{1 + AJLO \eta T_s (1 - \frac{1}{\eta})} \right\}.
\]

where \( h = 1/2 \) for BNFSK and \( h = 1 \) for BDPSK.

The SEP, which is equal to the probability that \( D < 0 \) when symbol 1 is transmitted, is obtained directly from the c.f. of \( D \) using the inversion theorem [10]. After changing the variable \( JLO \) to \( z = 4ju E_s N_0 \) in (3a), the SEP is given by

\[
P_e = - \frac{1}{(L-1)!} G'(L-1)(-1).
\]

where

\[
G(z)^{L-i} \frac{1}{z} = \frac{\psi_D(z/(4E_s N_0))}{z} = \exp \left\{ \sum_{k=1}^{L} \frac{b_k}{1 - z[1 + a_k]} - 1 \right\}.
\]

To obtain an expression for \( P_e \) we first evaluate the derivatives of \( H(z) = \ln G(z) \), and then apply Faa di Bruno’s formula [11] for the derivatives of a composite function to get \( G(z) \).

The mth derivative of \( H(z) \) is given by

\[
+ \frac{m - 1!}{2} \sum_{k=1}^{L} \frac{1 + a_k}{(1 - z[1 + a_k])^m} + \frac{m!}{2} \sum_{k=1}^{L} b_k \frac{1 + a_k}{(1 - z[1 + a_k])^{m+1}}.
\]
where the summation is over all \((L-1)\)-tuples \((h, \ldots, h')\) of integers in the range \([0, L-1]\) satisfying \(\sum h' = 1\). Applying (5), (6), and (3) in (4a), we obtain

\[
P_e = \text{exp} \left\{ \sum_{k=1}^{2L} \frac{E_{s}}{N_0} \sum_{m=1}^{L-1} \left( \frac{2 + 4h_k E_s}{2 + 4h_k E_s} \right)^m \right\}
\]

\[
\times \prod_{m=1}^{L-1} \sum_{\eta_k} \left[ \frac{1}{m!} \frac{1}{2r_m} \sum_{k=1}^{2L} \left( \frac{1 + 4h_k E_s}{2 + 4h_k E_s} \right)^m \right]^{1/m}
\]

where \(A_1, \ldots, A_{L-1}, B_1, \ldots, B_{L-1}\) and \(r_1, \ldots, r_{L-1}\) are derived from the fading statistics. Thus, (7) is a closed-form expression for the SEP for BNFSK and BDPSK signals over correlated Rician fading channels. To numerically compute the SEP, we simply need a precalculated lookup table of enumerations of the indices \(Z_1, \ldots, IL-I\) in the composite summation of (7).

III. NUMERICAL RESULTS

We present plots for the exponential correlation type of power balanced diversity, which arises, for example, in space diversity when the correlation between the diversity branches decreases with increase of branch spacing, and is characterized by

\[
K_{cc} = \frac{2}{a}\]

\[
K_{ss} = \begin{cases} 
\sigma^2_z, & \text{if } i = j \\
\rho^{1-l_1} \sigma^2_z, & \text{if } i \neq j
\end{cases}
\]

(8)

where \(a > 0\) and \(0 < p < 1\). We assume that there is a phase difference of \(\xi\) between the specular components of the \(A_k\) and \((k+1)\)th diversity branches, that is, for each \(k = 1, \ldots, L\)

\[
\left\{ \mu_{i} + \mu_{k} \right\} = \mu_{0} e^{(k-1)\xi}
\]

where \(\mu_{0}\) is a real constant.

It has been found from computations that for all \(\xi\) satisfying \(0 < |\xi| < \xi_{\text{max}}\), where \(\xi_{\text{max}}\) depends on the correlation coefficient \(p\), the diversity order \(L\), and the branch SNR \((E_s/N_0)/(|l + 2a|)\), the SEP increases with increase of the Rician factor \(K = \gamma/\gamma(2a)\), reaches a maximum at some \(K = K_{\text{max}}\), and then decreases as \(K\) increases. The value of \(K_{\text{max}}\) decreases as \(|\xi|\) goes from 0 to \(\xi_{\text{max}}\). For \(|\xi| > \xi_{\text{max}}\), the SEP attains a maximum at \(K = 0\), which corresponds to Rayleigh fading. Therefore, in the case of correlated branches, for a certain range of values of \(|\xi|\), the performance in Rayleigh fading is better than in Rician fading, and then decreases as \(|\xi|\) increases. In the case of independent branches where the performance in Rayleigh fading is worse than in Rician fading [5]. Fig. 1 shows the behavior of the performance in Rayleigh fading is worse than in Rician fading [5].
this behavior is nullified in $P_e$ as seen in [4, Figs. 1 and 2]. For BDPSK, the error performance of which is better than that of BNFSK by an SNR margin of 3 dB, plots of $P_e$ will exhibit similar behavior.

REFERENCES


