Evolution of the Intensity Profile of Cerenkov Second-Harmonic Radiation with Propagation Distance in Planar Waveguides

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Abstract—Using coupled mode theory, we have studied the output intensity profile of Cerenkov second-harmonic radiation from planar waveguides as a function of the propagation distance. In particular, we have obtained the variation of the intensity profile taking into account the effect of prism coupling as well as propagation loss, and have shown that the second-harmonic radiation evolves into a beam-like output. Results of the measured intensity profile of the second-harmonic radiation in proton-exchanged planar waveguides in Z-cut LiNbO3 are also presented that are consistent with the theory.

I. INTRODUCTION

There has recently been increasing interest in the study of nonlinear optical effects in an optical waveguide structure for realizing efficient functional devices such as parametric oscillators and frequency doublers (see, e.g., [1]). In particular, the second order effect enables one to realize a frequency-doubled coherent light source in the blue region by the phenomenon of second-harmonic generation (SHG) of near-infrared laser radiation using, for example, KDP, LiTaO3, or LiNbO3 as the nonlinear medium. Such short wavelength compact light sources are expected to find applications in high density optical data storage systems, efficient laser printers, laser television, etc. Use of a waveguide configuration for SHG offers several advantages including long interaction lengths, confinement of optical beams to small cross-sectional areas—leading to high power densities, and the possibility of employing unique phase matching techniques that are not applicable for bulk devices.

One of the simplest ways of obtaining efficient guided-wave SHG is using the Cerenkov configuration [2], wherein the “phase matching condition” is automatically satisfied. This eliminates the need for precise control of waveguide parameters, which is a stringent requirement for meeting the phase matching condition in other configurations. In the Cerenkov configuration, a part of the guided wave energy is frequency-doubled to generate a group of modes in the continuum of radiation modes. The output intensity profile of the second-harmonic radiation is a superposition of these radiation mode fields having propagation constants around the phase matching value. For practical applications, in addition to the conversion efficiency, it is important to know the output beam shape and the intensity profile of the second-harmonic radiation, since this defines the coupling mechanism to be employed to efficiently collect/utilize the radiation.

In this paper, we have used coupled-mode theory [3], [4] to study the evolution of the output intensity profile of the second-harmonic radiation in the Cerenkov configuration as a function of propagation distance in planar waveguides. In particular, we have shown that in practical waveguides having finite propagation loss, the intensity profile shows significant difference as compared to that in the case of lossless propagation. Our results for the intensity profile in the case of lossless propagation are consistent with the results of BPM calculations [7]. These results are also in agreement with the numerical simulation results of Hayata et al. [8].

To verify our theoretical model, we have also measured the intensity profile of the second-harmonic for various lengths of interaction in proton-exchanged planar waveguides in Z-cut LiNbO3. In the experimental set up, we have used a prism coupling arrangement to couple the pump beam into the waveguides. In the prism-film coupler, the coupling is nonuniform and takes place over a finite length, which is equal to the width of the incident beam [9]. For a comparison of the theoretical and the measured profiles in this case, we have also estimated the intensity profile of the second-harmonic by taking into account the input coupling at the base of the prism. With the combined effects of loss and prism coupling, the second-harmonic radiation evolves into a beam-like output having a near symmetric intensity profile. Results based on the simple theoretical calculations agree very well with the experimental measurements.

II. THEORY

Fig. 1 shows the waveguide configuration for Cerenkov SHG. The waveguide is assumed to support a single guided TM mode at the pump frequency ω. The pump wave generates a nonlinear polarization at 2ω, which then radiates energy into the continuum of radiation modes of the waveguide, automatically satisfying the phase matching condition: \( 2β(ω) = 2β'(ω) \), where \( β \) is the propagation constant. The total second-harmonic field at a particular value of \( z \) (i.e., propagation distance) is a superposition of radiation modes with propagation constants around the phase matching value \( 2β'(ω) \). The output intensity profile of the second-harmonic radiation can be obtained by calculating the \( \hat{e} \)-component of the Poynting vector \( S^{\hat{e}} \) as a function of the transverse...
coordinate $x$ (see Fig. 1). For TM modes, since $E_x, E_z$ are the only nonvanishing field components, $S^{(2)}_{1}^{(x)}(x)$ can be written as

$$S^{(2)}_{1}^{(x)}(x) = \frac{1}{2} \Re \left[ P^{(x)}_1(x) \times H^*(2\omega) \right]$$  \hspace{1cm} (1)

where $H^*(2\omega)$ is the total magnetic field at $2\omega$, and is expressed as

$$H^*(2\omega) = \int b_\omega(\beta_2, z) H_{2\omega}(\beta_2, z)e^{i(2\omega t - \beta_2 z)} d\beta_2,$$  \hspace{1cm} (2)

where $b_\omega$ is the $\omega$-dependent amplitude coefficient of a radiation mode having propagation constant $\omega$, and $H_{2\omega}$ is the field of the radiation mode; the asterisk on $H_\omega$ in (1) refers to the complex conjugate.

In the notation of various parameters in this paper, the subscript 1 (or 2) refers to fundamental (or second-harmonic) wavelength, subscript / (or 5) implies film (or substrate), and subscript o (or e) corresponds to the ordinary (or extraordinary) waves.

A. Evaluation of $S^{(2)}_{2}^{(x)}(x)$

To obtain an expression for $b_\omega$, we follow a coupled mode approach similar to that reported in [4] and get

$$62092, z) = \frac{-i(\omega)(P^{(x)})}{2\omega^2 e^2} \frac{d^2}{d\xi^2} \Delta \beta_\omega \sin \left( \frac{\Delta \beta_\omega}{2} \right),$$  \hspace{1cm} (3)

where $P^{(x)}$ is the power carried by the guided mode at $u$,

$$\Delta \beta = \beta_2 - 2\beta_1$$  \hspace{1cm} (4)

and $d$ is the overlap integral, defined as

$$d = \int_{-\infty}^{\infty} H^{(x)}_1(x) dx,$$  \hspace{1cm} (5)

where $P^{(x)}_B$ is the relevant nonlinear coefficient. Substituting for $b_\omega$ from (3) in (2), $H^{(x)}_2$ can be written as

$$H^{(x)}_2 = -i\frac{\partial P^{(x)}}{2\omega^2 e^2} e^{i(\omega t - \beta_1 z)}$$

\hspace{1cm}$\times$$\int H_{2\omega}(\beta_2, z) d\beta_2 \sin \left( \frac{\Delta \beta_\omega}{2} \right)e^{-i\beta_2 z/2} d\beta_2$

$$= K_0(a_o - ib_o) \quad \text{(say)},$$  \hspace{1cm} (6)

with

$$K_0 = -i\frac{\partial P^{(x)}}{2\omega^2 e^2} e^{i(\omega t - \beta_1 z)}$$  \hspace{1cm} (7)

$$ao = j H_{2\omega}(fa, x) \frac{1}{d\beta_2} \sin \left( \frac{\Delta \beta_\omega}{2} \right) d\beta_2,$$  \hspace{1cm} (8)

$$b_\omega = j \frac{1}{2} \frac{1}{H_{2\omega}(fa, x)} \frac{1}{d\beta_2} \sin \left( \frac{\Delta \beta_\omega}{2} \right) d\beta_2.$$  \hspace{1cm} (9)

From Maxwell's equations, for TM modes, $E_y$ can be expressed in terms of $H$ as

$$E_y = \frac{\partial H_z}{\partial t} \frac{1}{\epsilon_0}.$$  \hspace{1cm} (10)

Using (10) and (6) in (1), we get

$$S^{(2)}_{2}^{(x)}(x) = \frac{\beta_2^2}{4\omega \epsilon_0} \frac{1}{d\beta_2} \frac{1}{K_0} [e^2 + b_o^2].$$  \hspace{1cm} (11)

The above expression gives the $\omega$-component of the second-harmonic Poynting vector as a function of the transverse coordinate $x$, i.e., the intensity profile of the second-harmonic radiation, in terms of various waveguide, parameters.

B. Effect of Propagation Loss on $S^{(x)}(x)$

In Section II-A, we had assumed that the power $P^{(x)}$ in the guided mode is independent of the propagation distance $z$, i.e., we had neglected "pump depletion." However, practical waveguides having relatively large difference between the refractive indices of the guiding film and the substrate, such as proton exchanged waveguides in LiNbO$_3$, may exhibit several dB loss per centimeter length of the waveguide. In this case, even though the pump depletion due to conversion into second-harmonic is small, it is very important to consider the effect of propagation loss. We assume the loss of the fundamental mode (at $\omega$) to be of the form

$$p(x) = P^{(0)}(x) \exp(-a x),$$  \hspace{1cm} (12)

where $a$ is the attenuation coefficient. In this case, following the analysis given in [4], we get

$$\Delta \beta = \frac{K a}{\alpha^2 + \Delta \beta^2} \sin \left( \frac{\Delta \beta + i\alpha}{2} \right),$$  \hspace{1cm} (13)

where

$$K = \frac{-i \beta_0}{\omega \epsilon_0} e^{-(\beta_0 + \alpha z)},$$  \hspace{1cm} (14)

Substituting for $b_\omega$ from (13) in (2) and using (10), we get

$$S^{(2)}_{2}^{(x)}(x) = \frac{\beta_2^2 K^2}{4\omega \epsilon_0} (e^2 + b_o^2).$$  \hspace{1cm} (15)
where
\[ a = \int \frac{1}{(\alpha^2 + A/\Delta z)^2} C_{2d}H_{2d}(3x) \pi d \tau \]  
\[ b = \int \frac{1}{(\alpha^2 + \Delta \beta)^2} C_{2d}H_{2d}(p,2) \pi d \tau \]  
with
\[ C_1 = \cosh \left( \frac{\alpha z}{2} \right) \sin \left( \frac{\Delta \beta z}{2} \right) \]
\[ \cdot \left[ A/3 \right. \sin \left( \frac{\alpha z}{2} - A/3 \right) + \cos \left( \frac{\alpha z}{2} - A/3 \right) \]
\[ \cdot \left. \left[ -A/3 \sin \left( \frac{\alpha z}{2} - A/3 \right) + \cos \left( \frac{\alpha z}{2} - A/3 \right) \right] \right] \]
\[ C_2 = \cosh \left( \frac{\alpha z}{2} \right) \sin \left( \frac{\Delta \beta z}{2} \right) \]
\[ \cdot \left[ A/3 \sin \left( \frac{\alpha z}{2} - A/3 \right) + \cos \left( \frac{\alpha z}{2} - A/3 \right) \right] \]
\[ \cdot \left[ -A/3 \sin \left( \frac{\alpha z}{2} - A/3 \right) + \cos \left( \frac{\alpha z}{2} - A/3 \right) \right] \]  
\[ H_{2d}((3x) x) \] in both cases above (i.e., lossless and lossy wave guides) represents the radiation mode field for a given value of \( z \), the expressions for these are well-known [5], [6], and are given in the Appendix. Using these expressions, the integrals for \( a_0, a, b_0, \) and \( b \) can be evaluated.

### III. NUMERICAL RESULTS AND DISCUSSION

We consider a step index proton-exchanged LiNbO₃ planar waveguide with the following values of the various parameters:

\[ a^\wedge = 1.064 \text{ fim}, \quad P^\wedge = 10^5 \text{ W/m} \]
\[ d = 0.6 \text{ pin}, \quad A_n = 0.1 \]
\[ \eta_{loss} = n_{eff} = 2.232, \quad n_{ref} = 2.156, \quad n_{a_0} = 2.234, \quad n_{a_2} = 2.234, \quad d_{33} = -3.4 \times 10^{-12} \text{ m/V}. \]  

Fig. 2 shows the variation of the intensity of the second-harmonic as a function of the transverse coordinate \( x \) at interaction lengths \( z = 2, 5, \) and 8 mm, assuming no loss of the fundamental mode power (i.e., \( a = 0 \)). The peak at the extreme right in the intensity profile for a given value of \( z \) is the Cerenkov angle, which is given by \( \theta = \tan^{-1}(\gamma/\gamma) \), and is the same for all values of \( z \). \( x_p \) is the \( z \)-coordinate corresponding to the peak. It can be seen that for angles greater than the Cerenkov angle, there is no second-harmonic in the substrate; this is because of the destructive interference of the radiation mode fields in that region. This behavior is also consistent with the ray picture (see Fig. 1). The intensity profiles shown in Fig. 2 match well with those generated using the beam propagation method [7]. Since the amplitude coefficient of the second-harmonic is sharply peaked about \( (3z - 2/\gamma) \) [due to the “sine” function, cf. (3)], the integrals for \( a_0, a, b_0, \) and \( b \) are evaluated in the range \( 2/\gamma \pm 10 \). These integrals were numerically evaluated using standard subroutines on a computer; no appreciable change in the values of the integrals were observed by increasing the range of integration.

Fig. 3(a) and (b) show the intensity profiles of the second-harmonic radiation at different values of \( z \) for propagation losses of 1 and 10 dB/cm, respectively, in the same waveguide. As can be seen from the graphs, in the presence of a finite propagation loss, the intensity profile of the second-harmonic radiation is affected significantly. In a lossy waveguide, due to attenuation of the fundamental wave, the conversion efficiency per unit length decreases with \( z \). This results in a finite-size beam-like shape of the intensity profile (i.e., spatially limited in the transverse dimensions), which is unlike in the case of lossless propagation where the second-harmonic comes out as a line-source. A schematic representation of the evolution of the second-harmonic intensity profile with propagation is shown in Fig. 4.

### IV. MEASUREMENT OF THE INTENSITY PROFILE

To measure the intensity profile of the second-harmonic output, we fabricated proton-exchanged planar waveguides in Z-cut LiNbO₃ using benzoic acid melt. Results presented in the following correspond to a waveguide fabricated with an exchange time of 23 min at 240 °C. The waveguide was characterized by measuring the effective indices of the guided modes in a prism coupling arrangement. The waveguide supported a single TM mode at 1.064 μm and two modes at 0.543 μm. The waveguide ends were lapped and polished to enable measurements on Cerenkov radiation. A schematic representation of the experimental arrangement to measure the second-harmonic intensity profile is shown in Fig. 5. Pulsed output from a Q-
Fig. 3. $S_z$ vs. $x$ for lossy waveguides with (a) $\alpha = 1$ dB/cm, (b) $\alpha = 10$ dB/cm for $d = 0.6$ mm and $A_n = 0.1$.

Fig. 4. Schematic representation of the variation of the second-harmonic intensity profile at three different cross sections along the length of the waveguide. The broken-line curves correspond to the lossless case and the solid-line curves correspond to the intensity profile for lossy waveguides. The shaded profile represents the fundamental waveguide modes at $w$.

Fig. 5. Experimental set up for measuring the second-harmonic intensity profile. SMF is a single-mode fiber and FH is a fiber holder that is mounted on a translational stage.

Fig. 6. Typical plot of the measured SH intensity profile for an interaction length $h = 1.6$ mm. The crosses represent the measured points, which are joined by a smooth curve representing the profile.

switched Nd-YAG laser ($A = 1.064$ $\mu$m) is coupled into the waveguide using a prism coupling arrangement. The intensity profile of the second-harmonic at the edge of the substrate was scanned with a single mode fiber (core diameter $\sim 6$ $\mu$m), and the output of the fiber was fed to a detector through a filter. The output beam was scanned along the vertical direction, which was at about 26° with the (transverse) $a_1$-axis of the waveguide

This fact was taken into account in comparing the measured intensity profile with the calculated profile by changing the $x$ scale by a factor $l/(\cos 26°)$. The measurements were carried out for "interaction lengths" ($l_2$) of -1.0, 0.0, 0.8, 1.6, and 2.0 mm, respectively. The negative length refers to the case when the end of the prism lies beyond the end of the waveguide (see Fig. 5). Because of the finite width of the input beam, there is already second-harmonic generated in the waveguide-length behind the edge of the prism [4]. The actual interaction length is approximately the distance measured from the edge of prism (i.e., $l_2$) plus $h$, which is the input beam-width at the base of the prism.

Fig. 6 shows a typical measured intensity profile of the second-harmonic for $l_2 = 1.6$ mm, and Table I shows the measured full widths at half maxima of the intensity profiles for different values of $l_2$. From the table, we see that the beam width at the output end of the waveguide increases with the length of interaction, as expected. We may recall, however, that the intensity profile obtained in Section II exhibited a sharp fall at the transverse coordinate corresponding to the Cerenkov angle, whereas the measured intensity profile exhibits much
TABLE I

<table>
<thead>
<tr>
<th>h (mm)</th>
<th>Full width at half maximum (mm)</th>
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<tbody>
<tr>
<td>-1.0</td>
<td>0.09</td>
</tr>
<tr>
<td>0.0</td>
<td>0.11</td>
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<tr>
<td>0.8</td>
<td>0.13</td>
</tr>
<tr>
<td>1.6</td>
<td>0.15</td>
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<tr>
<td>2.0</td>
<td>0.19</td>
</tr>
</tbody>
</table>

reduced asymmetry. We shall show in the next section that this difference is due to the combined effects of the waveguide loss and the prism-coupling arrangement used for pump beam coupling into the waveguide. The input beam at the base of the prism (and hence the energy distribution) is nearly Gaussian, and coupling takes place over a length \( \Lambda \), which is of the order of a millimeter; whereas in the theory we had assumed that the input power \( P_i \) is coupled at \( z = 0 \) (which corresponds to the case of end fire coupling). Consequently, the peak in the intensity profile shifts towards the waveguide interface resulting in a near-symmetric profile.

Fig. 7 show the photographs of the intensity distribution for \( l_1 = 1.0, 2.0, \) and 4.0 mm, taken at a distance of 80 cm from the waveguide end. One can clearly see that as the length of interaction increases the beam width in the far-field decreases; this should be expected because with the increase in length of interaction, the beam at the exit face of the substrate broadens, and a broader beam diffracts to a smaller extent with propagation. We can also notice the secondary lobes of diffraction for the case shown in Fig. 7(c) which corresponds to the largest interaction length. The diffraction pattern resembles that of a single slit, indicating that the radiation is in the form of a well confined beam. Fig. 8 shows the measured intensity profile corresponding to Fig. 7(a) at the same distance of 80 cm. The corresponding FWHM is 3.03 mm. The measurement was carried out by a pinhole-masked large-area PIN detector, which was mounted on a motorized scanner. The output of the detector was directly connected to an X–Y plotter. The intensity profile appears nearly symmetric and is almost similar to the intensity profile measured at the waveguide end (see Fig. 6). We may mention here that the field distribution of the second-harmonic at the exit end of the waveguide, which is a superposition of the radiation modes, is already in the "far-field region." Hence, even at a distance of 80 cm away from the waveguide-end, the shape of the intensity profile is expected to be similar. Indeed, we have actually verified this by taking the Fourier transform of the calculated intensity profile at the exit end of the waveguide.

V. EFFECT OF PRISM COUPLING ARRANGEMENT AND PROPAGATION LOSS ON THE INTENSITY PROFILE OF THE SECOND-HARMONIC

In the discussions in Sections II-A and B, the entire fundamental mode power \( P_M \) was assumed to be coupled at \( z = 0 \). This situation corresponds to the case of end fire coupling. On the other hand, in a prism-film coupler the input wave energy is coupled into the film over a certain propagation distance \( h \) (under the prism base) depending on the input beam width, and the energy confined to the film propagates beyond \( z = h \) (see Fig. 5) with certain propagation loss. Thus, the power in the fundamental mode \( P(z) \) increases in the region \( 0 < z < l_1 \) and then decays exponentially (due to propagation loss) beyond \( z = \Lambda \). In this section, making a few simplifying assumptions, we derive an expression for \( S_{2}^{(2\omega)}(z) \) taking into account the nonuniform coupling at the base of the prism.

The dependence of coupled power on the propagation distance \( z \) (under the prism), in a prism-film coupler arrangement, was obtained by Tien [9] in the form

\[ P(z) \sim \frac{(1 - e^{-a_1 z})^2}{G L z}, \quad (21) \]

where \( a_1 \) is a parameter specifying the coupling strength. Since the behavior of the functions \( (1 - e^{-z^2/\lambda}) \) and \( (1 - e^{-z^2}) \) are approximately same, for mathematical simplicity, we assume that the variation of the coupled power up to the edge of the prism is given by

\[ P''(z) = P_0' \frac{P'(1 - e^{-a_1 z})}{G L}, \quad z < l_1, \quad (22) \]

where \( P' = \tau' A_{0} l_{1}^{-1} \) is a constant chosen to satisfy the continuity of \( P'' \) at \( z = l_1 \) and \( \Lambda \) is the width of the input beam at the base of the prism. If we also take into account the propagation loss of the fundamental mode in the region of prism coupling [cf. (12)], then the variation of \( P''(z) \) can be written as

\[ P''(z) = P_0' \frac{P'(1 - e^{-a_1 z})}{G L z}, \quad z < l_1, \quad (23) \]

where \( a_1 \) is the attenuation coefficient of the fundamental mode. For optimum input coupling, it can be shown that \( (a_1 + a) l_1 /\Lambda = 1.25 \). Hence, when the coupling is maximized experimentally, we may assume that \( (a_1 + a) l_1 = 1.25 \). Substituting for \( P''(z) \) from (23) and again following coupled mode theory, we obtain the second-harmonic amplitude coefficient as

\[ b_2 = \frac{1}{4} \int_{l_1}^{l_0} e^{i A_3 z} (1 - e^{-i (a_1 + a) z}) dz + C \int_{l_1}^{l_0} e^{i (\alpha_0 - a_3) z} dz \]

\[ = C \left[ e^{i A_3 l_0} - \frac{C e^{i A_3 z}}{i A_3 - (\alpha_0 - a_3)} \right] \left( e^{i (\alpha_0 - a_3) l_0} - 1 \right) \left( e^{i (\alpha_0 - a_3) l_0} - 1 \right) \right], \quad (24) \]

where \( C = \sqrt{\tau' A_{0} l_{1}^{-1}} \) and \( C' = C P' \). The Poynting vector in this case is given as

\[ S_{2}^{(2\omega)}(z) = \left| \beta_2 \frac{d}{d\omega} \left( \int_{l_1}^{l_0} b_2(z) \right) d\omega \right|^2, \quad (25) \]

which can be evaluated numerically. Thus, (25) would give the intensity profile of the second-harmonic at any \( z \) when the pump beam is coupled into the waveguide using a prism coupling arrangement.

Fig. 9 shows the second-harmonic intensity profile with the combined effect of the prism coupling arrangement and propagation loss. One can notice the slow drop in the power.
Fig. 7. Photographs of the far field of the Cerenkov second-harmonic radiation at interaction lengths (h) of (a) 1.0 mm, (b) 2.0 mm, and (c) 4.0 mm.

Fig. 8. Plot of the measured intensity profile corresponding to Fig. 7(a) at a distance 80 cm away from the waveguide end. 

Fig. 9. $\chi^{(2)}$ vs. x for lossy waveguides, taking into account the prism coupling for $a = 10 \text{ dB/cm}$, $d = 0.6 \text{ mm}$, and $\Delta n = 0.1$. 

for $x > x_p$ ($x_p$ refers to the peak position corresponding to the Cerenkov angle) as compared to the corresponding plots in Fig. 3(a) and (b). As discussed above, this is due to the finite propagation distance over which energy couples into the waveguide in a prism-coupling arrangement. The combined effect of loss and prism coupling arrangement could result in a nearly symmetric beam shape.

Fig. 10 shows the best-fitting calculated intensity profile along with the measured values for an interaction length $h = 1.6$ mm (cf. Fig. 5). The theoretical fitting was done by choosing suitable values of the parameters $a$ and $l_x$; in this case $a$ was found to be 35 dB/cm and $l_x = 0.7$ mm. As can be seen from the figure, the fitting is very good except for a slight mismatch near the surface of the waveguide (x m 0). Similar fitting was also achieved for other interaction lengths. The mismatch near x = 0 could be because of rounding off of the waveguide end near the surface during the end-polishing process, which has smoothened the measured profile. We may mention here that the loss figure obtained by fitting is relatively high, which may be due to the fact that we were using unannealed waveguides, and without dilute melt proton
exchange. Also, due to coupling of Q-switched Nd:YAG laser light, the waveguides were seen to undergo photorefractive damage. This was also supported by the fact that no visible streak (due to scattered light) corresponding to guided modes appeared on the surface of the waveguide when light from a He-Ne laser was coupled into the waveguides after observing SHG using the Nd:YAG laser, whereas good streaks were visible before coupling light from the Nd: YAG laser. This is being investigated separately.

VI. CONCLUSION

We have studied the evolution of the output intensity profile of the second-harmonic radiation in the Cerenkov configuration as a function of the propagation distance in planar waveguides. Using coupled mode theory, we have derived an expression for second-harmonic intensity as a function of the transverse waveguide coordinate and the propagation distance. We have obtained the intensity profile taking into account the effect of prism coupling as well as loss, and have shown that the second-harmonic radiation evolves into a confined beam-like output, unlike in the case of lossless propagation. We have also presented results of the measured intensity profile of the second-harmonic radiation corresponding to proton-exchanged LiNbO₃ waveguides, which are consistent with the theoretical calculations.

APPENDIX

The expressions for the TM radiation modes are given by [5]

\[
H_{2m}(x) = \begin{cases} 
B_e \exp \left( A_m x \right) & x < 0 \\
B_e \left( \cos \sigma_m x + n_{2e}^2 \frac{\mathrm{d} \sigma_m}{\mathrm{d} n_{2e}^2} \sin \sigma_m x \right) & 0 < x < d \\
B_e \left( \cos \sigma_m x + n_{2e}^2 \frac{\mathrm{d} \sigma_m}{\mathrm{d} n_{2e}^2} \sin \sigma_m x \right) & x > d 
\end{cases}
\]

where

\[
\sigma_m^2 = \frac{n_{2e}^2}{n_{2e}^2} \left( \frac{\pi}{\varepsilon_{2e} n_{2e}} \right)^2 \left( \frac{\varepsilon_{2e} n_{2e}}{\varepsilon_{2e}} \right)^2, \quad \sigma_m^2 = \frac{n_{2e}^4}{n_{2e}^2} \left( \frac{\pi}{\varepsilon_{2e} n_{2e}} \right)^2 \left( \frac{\varepsilon_{2e} n_{2e}}{\varepsilon_{2e}} \right)^2
\]

\[
\Delta_m^2 = \left( \frac{\sigma_m^2}{\sigma_m^2} \right)
\]

\[
B_m = \left\{ \begin{array}{l}
\frac{2}{n_{2e}^2} \left( 2m + 1 \right) \sigma_m^2 + \left( \frac{\varepsilon_{2e} n_{2e}}{\varepsilon_{2e}} \right)^2

\end{array} \right\}
\]

The rest of the parameters are defined in the text.

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